TENSORIAL NORM INEQUALITIES FOR TAYLOR'S EXPANSIONS OF FUNCTIONS OF SELFADJOINT OPERATORS IN HILBERT SPACES

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Abstract. Let H be a Hilbert space. In this paper we show among others that, if f is of class C^{n+1} on the open interval I, P and Q are selfadjoint operators with $\mathrm{Sp}(P)$, $\mathrm{Sp}(Q) \subset I$ and if $\left\| f^{(n+1)} \right\|_{I,\infty} := \sup_{u \in I} \left| f^{(n+1)}(u) \right| < \infty$, then

$$\left\| f(P) \otimes 1 - \sum_{k=0}^{n} \frac{1}{k!} (P \otimes 1 - 1 \otimes Q)^k \left(1 \otimes f^{(k)}(Q) \right) \right\|$$

$$\leqslant \frac{1}{(n+1)!} \| P \otimes 1 - 1 \otimes Q \|^{n+1} \left\| f^{(n+1)} \right\|_{I,\infty}.$$

If $\left| f^{(n+1)} \right|$ is convex on I, then also

$$\left\| f(P) \otimes 1 - \sum_{k=0}^{n} \frac{1}{k!} (P \otimes 1 - 1 \otimes Q)^k \left(1 \otimes f^{(k)}(Q) \right) \right\|$$

$$\leqslant \frac{1}{(n+1)!} \| P \otimes 1 - 1 \otimes Q \|^{n+1} \left[\frac{\left\| f^{(n+1)}(P) \right\| + (n+1) \left\| f^{(n+1)}(Q) \right\|}{n+2} \right].$$

Several examples for fundamental functions such as the logarithm and exponential are also provided.

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