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ON THE β–MODIFICATION OF THE MELLIN–GAUSS–WEIERSTRASS KERNEL AND ITS RELATED INFORMATION POTENTIAL

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Abstract. In the current study, we investigate the behaviour of β-modification of the Mellin-Gauss-Weierstrass (MGW) type operators with respect to pointwise and uniform convergence. Moreover, we give a Voronovskaya approximation formula for the MGW type operators using the new kernel. This formula contains Mellin derivatives and a different notion of moment which was called the logarithmic moment. In the last part, we analyze the related information potential, the variance $V[logp(\cdot,\cdot)]$ and expected value $EV[logp(\cdot,\cdot)]$ using the modified MGW kernel $p(\cdot,\cdot)$.

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Paper JMI-19-02

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