ON C–HYPONORMAL OPERATORS

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Abstract. A bounded linear operator $T : \mathscr{H} \to \mathscr{H}$ is a *C-hyponormal* operator if $T^*T - CTT^*C \geqslant 0$ for a conjugation C on $\mathscr{H}$. In this paper, we study properties of C-hyponormal operators. Especially, we prove that for $\mathscr{M} \in Lat(T)$ and a conjugation $C = C_1 \oplus C_2$ on $\mathscr{H} = \mathscr{M} \oplus \mathscr{M}^\perp$, if T is C-hyponormal, then $T|_{\mathscr{M}}$ is C_1-hyponormal. Moreover, we show that $T - \lambda I$ is C-hyponormal for all $\lambda \in \mathbb{C}$ if and only if T is a complex symmetric operator. Finally, we prove that if T^* is p-hyponormal for $0 < p \leqslant 1$ and C is a conjugation on $\mathscr{H}$, then T is C-hyponormal if and only if T is normal.

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REFERENCES

[1] C. BENHIDA, M. CHO, E. KO, AND J. LEE, *On symmetric and skew-symmetric operators*, Filomat **32**: 1 (2018), 293–303.

[2] G. CASSINELLI, E. VITO, A. LEVRERO, AND P. J. LAHTI, *The Theory of Symmetry Actions in Quantum Mechanics*, Lecture Notes in Physics **654**, Springer, 2004.

[3] R. G. DOUGLAS, *On Majorization, Factorization, and Range Inclusion of Operators on Hilbert Space*, Proc. Amer. Math. Soc. **17** (2) (1966), 413–415.

[4] C. FOIAŞ AND A. E. FRAZHO, *The Commutant Lifting Approach to Interpolation Problems*, Operator Theory: Advances and Applications **44**, Birkhäuser, 1990.

[5] S. R. GARCIA AND M. PUTINAR, *Complex symmetric operators and applications*, Trans. Amer. Math. Soc. **358** (2006), 1285–1315.

[6] S. R. GARCIA, D. E. POORE, AND J. E. TENER, *Unitary equivalence to a complex symmetric matrix: Low dimensions*, Linear Algebra Appl. **437** (2012), 271–284.

[7] E. KO AND J. E. LEE, *Remark on complex symmetric operator matrices*, Linear Multilinear Algebra. **67** (9) (2019), 1198–1216.

[8] E. KO, J. E. LEE, AND M. LEE, *On properties of C-normal operators*, Banach J. Math. Anal. **15** (2021), Article 65.

[9] E. KO, J. E. LEE, AND M. LEE, *On properties of C-normal operators II*, Annals of Functional Analysis **14** (2023), Article 29.

[10] M. PTAK, K. SIMIK, AND A. WICHER, *C-normal operators*, Electronic J. Linear Alg. **36** (2020), 67–79.

[11] C. WANG, J. ZHAO, AND S. ZHU, *Remark on the structure of C-normal operators*, Linear and Multi. Alg. **70** (9) (2022), 1682–1696.

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