

SHARP INEQUALITIES FOR ZALCMAN FUNCTIONAL OF LOGARITHMIC COEFFICIENTS OF INVERSE FUNCTIONS IN CERTAIN CLASSES OF ANALYTIC FUNCTIONS

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Abstract. We study Hankel matrices whose entries are logarithmic coefficients of inverse functions in selected subclasses of analytic functions. Particularly, we give sharp bounds for the second Hankel determinant which reduces to Zalcman functional of logarithmic coefficients of inverse convex and starlike functions, as well as of functions of bounded turning.

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