

RATES OF CONVERGENCE FOR ITERATES OF POSITIVE LINEAR OPERATORS

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Abstract. We are concerned with positive linear operators defined on $C(X)$, where X is a simplex or a hypercube. We assume that the operators preserve the affine functions. After identifying an eigenvalue $a \in [0, 1)$ of such an operator L , we show that the sequence $(L^k f)_{k \geq 1}$ has a limit Vf , $f \in C(X)$ and $|L^k f(x) - Vf(x)|$ is dominated by a^k multiplied by a factor depending on L , f and x . These general results are applied to several classical or recently introduced operators acting on simplices and hypercubes.

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REFERENCES

- [1] A. M. ACU, G. BASCANBAZ-TUNCA, I. RASA, *Differences of positive linear operators on simplices*, J. Funct. Spaces, Article Number 5531577, 2021.
- [2] A. M. ACU, M. HEILMANN, I. RASA, *Eigenstructure and iterates for uniquely ergodic Kantorovich modifications of operators II*, Positivity **25** (2021), 1585–1599.
- [3] A. M. ACU, M. HEILMANN, I. RASA, *Iterates of convolution-type operators*, Positivity **25** (2021), 495–506.
- [4] O. AGRATINI, R. PRECUP, *Iterates of multidimensional approximation operators via Perov theorem*, Carpathian J. Math. **38** (3) (2022), 539–546.
- [5] O. AGRATINI, I. RUS, *Iterates of a class of discrete linear operators via contraction principle*, Commentat. Math. Univ. Carol. **44** (3) (2003), 555–563.
- [6] F. ALTOMARE, M. CAMPITI, *Korovkin-type Approximation Theory and its Applications*, Series: De Gruyter Studies in Mathematics, **17**, 1994.
- [7] F. ALTOMARE, M. CAPPELLETTI MONTANO, V. LEONESSA, I. RAȘA, *Markov Operators, Positive Semigroups and Approximation Processes*, Walter de Gruyter, Berlin, Munich, Boston (2014).
- [8] C. BADEA, *Bernstein polynomials and Operator Theory*, Result. Math. **53** (2009), 229–236.
- [9] L. BEUTEL, H. GONSKA, D. KACSÓ AND G. TACHEV, *Variation-diminishing splines revised*, in Proc. Int. Sympos. on Numerical Analysis and Approximation Theory (Radu Trâmbițaș, ed.), Presa Universitară Clujeană, Cluj-Napoca, 2002, 54–75.
- [10] M. M. BIROU, *Rates of convergence for the iterates of some positive linear operators*, Mediterr. J. Math. **14**, 129 (2017).
- [11] A. DORAI, *A Korovkin-type theorem for sequences of positive linear operators on function spaces*, Positivity **25** (2021), 2017–2027.
- [12] I. GAVREA, M. IVAN, *Asymptotic behaviour of the iterates of positive linear operators*, Abstr. Appl. Anal. **11** Art. ID 670509 (2011).
- [13] H. GONSKA, *Quantitative Aussagen zur Approximation durch positive lineare Operatoren*, Dissertation, Universität Duisburg 1979.
- [14] H. GONSKA, D. KACSÓ, P. PIȚUL, *The degree of convergence of over-iterated positive linear operators*, J. Appl. Funct. Anal. **1** (4) (2006), 403–423.
- [15] H. GONSKA, R. PĂLTĂNEA, *Quantitative convergence theorems for a class of Bernstein-Durrmeyer operators preserving linear functions*, Ukrainian Math. J. **62** (2010), 913–922.

- [16] H. GONSKA, R. PĂLTĂNEA, *Simultaneous approximation by a class of Bernstein-Durrmeyer operators preserving linear functions*, Czechoslovak Math. J. **60** (135) (2010), 783–799.
- [17] H. GONSKA, I. RAŞA, *On infinite products of positive linear operators reproducing linear functions*, Positivity (2013), **17**: 67–79.
- [18] T. N. T. GOODMAN, A. SHARMA, *A modified Bernstein-Schoenberg operator*, Proc. of the Conference on Constructive Theory of Functions, Varna 1987 (ed. by Bl. Sendov et al.), 166–173, Sofia: Publ. House Bulg. Acad. of Sci. 1988.
- [19] G. GWÓZDŹ-LUKAWSKA, J. JACHYMSKI, *IFS on a metric space with a graph structure and extensions of the Kelisky-Rivlin theorem*, J. Math. Anal. Appl. **356** (2), 453–463 (2009).
- [20] M. HEILMANN, I. RAŞA, *Eigenstructure and iterates for uniquely ergodic Kantorovich modifications of operators*, Positivity **21** (2017), 897–910.
- [21] J. JACHYMSKI, *Convergence of iterates of linear operators and the Kelisky-Rivlin type theorems*, Studia Math. **195** (2) (2009), 99–112.
- [22] A. LUPAŞ, *Die Folge der Betaoperatoren*, Dissertation, Universität Stuttgart, 1972.
- [23] R. PĂLTĂNEA, *A class of Durrmeyer type operators preserving linear functions*, Ann. Tiberiu Popoviciu Sem. Funct. Equat. Approxim. Convex. (Cluj-Napoca) **5** (2007), 109–117.
- [24] I. RAŞA, *Asymptotic behaviour of iterates of positive linear operators*, Jaen J. Approx. **1** (2) (2009), 195–204.
- [25] I. RAŞA, *CO -semigroups and iterates of positive linear operators: asymptotic behaviour*, Rend. Circ. Mat. Palermo **2** (Suppl 82) (2010), 123–142.