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GENERALIZED INTEGRATION OPERATORS FROM THE BESOV SPACE INTO GENERAL FUNCTION SPACES

XIANGLING ZHU* AND DAN QU

Abstract. The boundedness of the inclusion mapping from the Besov space B_p into a class of the tent type space $\mathcal{T}_s^{p,n}(\mu)$ is studied. As an application, the boundedness, compactness and essential norm of the generalized integral operators $T_g^{n,k}$ and $S_g^{n,0}$ from the Besov space B_p to general function spaces are also investigated.

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