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THE C–N–STAR, S–STAR AND C–MINUS PARTIAL ORDERS

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Abstract. In this paper, we derive several characterizations of the s-star partial order in terms of the core-nilpotent decomposition, and establish the conditions under which the s-star partial order implies the C-N-star partial order. By applying the core-EP decomposition, we introduce a new partial order, the c-minus partial order, which generalizes the core-minus partial order. Additionally, we provide several characterizations and properties of the c-minus partial order.

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