UPPER BOUND ESTIMATE FOR THE NORM OF
REPEATED DE LA VALLÉE POUSSIN OPERATORS

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Abstract. The Lebesgue constant for the repeated de la Vallée Poussin operator, defined in the space of continuous periodic functions, is studied. An integral representation of the repeated de la Vallée Poussin means is obtained as a sum of Riemann integrals over finite domains. Based on this, an upper bound for the norm of the repeated de la Vallée Poussin operators is derived, expressed in terms of the well-studied Lebesgue constant of the Fourier operator.

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