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ON NEWTON–LIKE INEQUALITIES FOR COMPLEX NUMBERS

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Abstract. I. Newton famously stated that the sequence of normalized elementary symmetric polynomials has the following property: the square of each polynomial is greater than or equal to the product of its two adjacent polynomials when evaluated at any real numbers. We introduce several novel generalizations of this property for evaluations on multisets of self-conjugate complex numbers in the angular sector $|\arg z| \leqslant \pi/4$.

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