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P–NUMERICAL RADIUS INEQUALITIES FOR 2×2 OPERATOR MATRICES

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Abstract. In this paper, we present a refinement of the triangle inequality for Schatten p-norm, and specific example is given to compare our result with the triangle inequality for Schatten p-norm. As an application, a new lower bound for p-numerical radius is obtained. In addition, some bounds for p-numerical radius of 2×2 operator matrices are established, which extend the results of previous studies. Moreover, Schatten p-norm equalities of 2×2 operator matrices are also given.

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