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ON A RECURSIVE RELATION AND ITS CONNECTIONS TO NUMBER THEORY

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Abstract. We give some theoretical explanations for solving the recursive relation

$$1+\sum_{d|n}(-1)^{\frac{n}{d}}a_d=0,$$

by finding its connections with number theory.

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REFERENCES

[1] V. ABRAMOVICH, *Zadacha M624*, (5) (1980), 36, (in Russian).
[2] V. ABRAMOVICH, *Reshenie Zadachi M624*, (3) (1981), 27–28, (in Russian).
[3] I. BAJO AND E. LIZ, *Global behaviour of a second-order nonlinear difference equation*, J. Differ. Equat. Appl. **17** (10) (2011), 1471–1486.
[4] M. I. BASHMAKOV, B. M. BEKKER AND V. M. GOL'HOVOI, *Zadachi po Matematike. Algebra i Analiz*, Nauka, Moskva, 1982, (in Russian).
[5] L. BERG AND S. STEVIĆ, *On the asymptotics of the difference equation* $y_n(1+y_{n-1}\cdots y_{n-k+1})=y_{n-k}$, J. Difference Equ. Appl. **17** (4) (2011), 577–586.
[6] D. BERNOULLI, *Observationes de seriebus quae formantur ex additione vel substractione quacunque terminorum se mutuo consequentium, ubi praesertim earundem insignis usus pro inveniendis radicum omnium aequationum algebraicarum ostenditur*, Commentarii Acad. Petropol. III, 1728 (1732), 85–100, (in Latin).
[7] B. P. DEMIDOVICH AND I. A. MARON, *Computational Mathematics*, Mir Publishers, Moscow, 1973.
[8] A. DE MOIVRE, *Miscellanea analytica de seriebus et quadraturis*, J. Tonson & J. Watts, Londini, 1730, (in Latin).
[9] T. FORT, *Finite Differences and Difference Equations in the Real Domain*, Oxford Univ. Press, London, 1948.
[10] C. JORDAN, *Calculus of Finite Differences*, Chelsea Publishing Company, New York, 1965.
[11] V. A. KRECHMAR, *A Problem Book in Algebra*, Mir Publishers, Moscow, 1974.
[12] S. F. LACROIX, *Traité des Differénces et des Séries*, J. B. M. Duprat, Paris, 1800, (in French).
[13] J.-L. LAGRANGE, *OEuvres*, t. II, Gauthier-Villars, Paris, 1868, (in French).
[14] P. S. LAPLACE, *Recherches sur l'intégration des équations différentielles aux différences finies et sur leur usage dans la théorie des hasards*, Mémoires de l' Académie Royale des Sciences de Paris 1773, t. VII, (1776) (Laplace OEuvres, VIII, 69–197, 1891), (in French).
[15] L. M. MILNE-THOMSON, *The Calculus of Finite Differences*, MacMillan and Co., London, 1933.
[16] D. S. MITRINOVIĆ AND D. D. ADAMOVIĆ, *Nizovi i Redovi*, Naučna Knjiga, Beograd, Serbia, 1980, (in Serbian).
[17] D. S. MITRINOVIĆ AND J. D. KEČKIĆ, *Metodi Izračunavanja Konačnih Zbirova*, Naučna Knjiga, Beograd, 1984, (in Serbian).
[18] G. PAPASCHINOPOULOS AND C. J. SCHINAS, *Invariants for systems of two nonlinear difference equations*, Differ. Equ. Dyn. Syst. **7** (1999), 181–196.

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[19] G. PAPASCHINOPOULOS AND C. J. SCHINAS, *Invariants and oscillation for systems of two nonlinear difference equations*, Nonlinear Anal. Theory Methods Appl. **46** (2001), 967–978.

[20] G. PAPASCHINOPOULOS AND C. J. SCHINAS, *On the dynamics of two exponential type systems of difference equations*, Comput. Math. Appl. **64** (2012), 2326–2334.

[21] G. PAPASCHINOPOULOS, C. J. SCHINAS, AND G. STEFANIDOU, *On a k-order system of Lyness-type difference equations*, Adv. Difference Equ. **2007**, Article No. 31272, (2007).

[22] G. PAPASCHINOPOULOS AND G. STEFANIDOU, *Asymptotic behavior of the solutions of a class of rational difference equations*, Inter. J. Difference Equations **5** (2) (2010), 233–249.

[23] G. POLYA AND G. SZEGÖ, *Problems and Theorems in Analysis II*, Springer, Berlin, 1976.

[24] M. H. RHOUMA, *The Fibonacci sequence modulo π, chaos and some rational recursive equations*, J. Math. Anal. Appl. **310** (2005), 506–517.

[25] J. RIORDAN, *Combinatorial Identities*, John Wiley & Sons Inc., New York-London-Sydney, 1968.

[26] C. SCHINAS, *Invariants for difference equations and systems of difference equations of rational form*, J. Math. Anal. Appl. **216** (1997), 164–179.

[27] S. STEVIĆ, *On the recursive sequence $x_{n+1} = \alpha_n + (x_{n-1}/x_n)$ II*, Dyn. Contin. Discrete Impuls. Syst. **10a** (6) (2003), 911–916.

[28] S. STEVIĆ, *Boundedness character of a class of difference equations*, Nonlinear Anal. TMA **70** (2009), 839–848.

[29] S. STEVIĆ, *Difference equations and nonexistence of solutions of an integral equation*, Appl. Math. Comput. **219** (2012), 1998–2003.

[30] S. STEVIĆ, *On the system of difference equations $x_n = c_n y_{n-3}/(a_n + b_n y_{n-1} x_{n-2} y_{n-3})$, $y_n = \gamma_n x_{n-3}/(\alpha_n + \beta_n x_{n-1} y_{n-2} x_{n-3})$*, Appl. Math. Comput. **219** (2013), 4755–4764.

[31] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *On some solvable difference equations and systems of difference equations*, Abstr. Appl. Anal. Vol. 2012, Article ID 541761, (2012), 11 pp.

[32] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *On a third-order system of difference equations with variable coefficients*, Abstr. Appl. Anal. Vol. 2012, Article ID 508523, (2012), 22 pages.

[33] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *On a solvable system of rational difference equations*, J. Difference Equ. Appl. **20** (5–6) (2014), 811–825.

[34] S. STEVIĆ, J. DIBLIK, B. IRIČANIN AND Z. ŠMARDA, *Solvability of nonlinear difference equations of fourth order*, Electron. J. Differential Equations. Vol. 2014, Article No. 264, (2014), 14 pages.

[35] S. STEVIĆ, B. IRIČANIN AND W. KOSMALA, *More on a hyperbolic-cotangent class of difference equations*, Math. Methods Appl. Sci. **42** (2019), 2974–2992.

[36] S. STEVIĆ, B. IRIČANIN, W. KOSMALA AND Z. ŠMARDA, *Note on the bilinear difference equation with a delay*, Math. Methods Appl. Sci. **41** (2018), 9349–9360.

[37] S. STEVIĆ, B. IRIČANIN AND Z. ŠMARDA, *On a product-type system of difference equations of second order solvable in closed form*, J. Inequal. Appl. Vol. 2015, Article No. 327, (2015), 15 pages.

[38] S. STEVIĆ, B. IRIČANIN AND Z. ŠMARDA, *Two-dimensional product-type system of difference equations solvable in closed form*, Adv. Difference Equ. Vol. 2016, Article No. 253, (2016), 20 pages.

[39] I. M. VINOGRADOV, *Osnovy Teorii Chisel*, Izdanie Devyatoe, Nauka, Moskva, 1981, (in Russian). English translation: An Introduction to the Theory of Numbers, Pergamon Press, London & New York, 1955.