Journal of Mathematical Inequalities
Volume 19, Number 2 (2025), 637–647
doi:10.7153/jmi-2025-19-39

NOTE ON THE NORM OF LINEAR COMBINATIONS OF A CLASS OF LINEAR OPERATORS

STEVO STEVIĆ

Abstract. Let $H(\mathbb{D})$ be the space of analytic functions on the open unit disk $\mathbb{D}$ in the complex plane $\mathbb{C}$, $S(\mathbb{D}) = \{f \in H(\mathbb{D}) : f(\mathbb{D}) \subseteq \mathbb{D}\}$, $\mathscr{K}(\mathbb{D})$ the space of Cauchy transforms on $\mathbb{D}$, $\mathscr{W}_w^{(m)}(\mathbb{D})$ the mth weighted type space on $\mathbb{D}$ with the weight function w and $m \in \mathbb{N}_0$, $\mathscr{W}_{w,0}^{(m)}(\mathbb{D})$ the little mth weighted type space on $\mathbb{D}$, and

$$L_s f(z) = \sum_{k=1}^{s} \alpha_k \sum_{j=0}^{n_k} u_{j,k}(z) f^{(j)}(\varphi_{j,k}(z)), \quad z \in \mathbb{D},$$

where $s \in \mathbb{N}$, $n_k \in \mathbb{N}_0$, $k = \overline{1,s}$, $\alpha_k \in \mathbb{C}$, $k = \overline{1,s}$, $u_{j,k} \in H(\mathbb{D})$, $j = \overline{0,n_k}$, $k = \overline{1,s}$, and $\varphi_{j,k} \in S(\mathbb{D})$, $j = \overline{0,n_k}$, $k = \overline{1,s}$. We find the norm of the operator $L_s : \mathscr{K}(\mathbb{D}) \to \mathscr{W}_w^{(m)}(\mathbb{D})$ in terms of above-mentioned parameters and symbols, and present a characterization for the boundedness of the operator $L_s : \mathscr{K}(\mathbb{D}) \to \mathscr{W}_{w,0}^{(m)}(\mathbb{D})$, considerably extending several recent results in the literature.

Mathematics subject classification (2020): Primary 47B38.
Keywords and phrases: Product type operator, spaces of analytic functions, open unit disk, norm of an operator.

REFERENCES

[1] L. V. AHLFORS, *Complex Analysis*, Third Edition, McGraw-Hill, Inc. New York et al, 1979.
[2] C. COWEN AND B. MACCLUER, *Composition Operators on Spaces of Analytic Functions*, CRC Press, Boca Raton, 1995.
[3] J. DU, S. LI AND Y. ZHANG, *Essential norm of generalized composition operators on Zygmund type spaces and Bloch type spaces*, Annal. Polon. Math. **119** (2) (2017), 107–119.
[4] Z. GUO, L. LIU AND Y. SHU, *On Stević-Sharma operators from the mixed-norm spaces to Zygmund-type spaces*, Math. Inequal. Appl. **24** (2) (2021), 445–461.
[5] R. A. HIBSCHWEILER AND T. H. MACGREGOR, *Fractional Cauchy Transforms*, Chapman & Hall/CRC, Boca Raton, London, New York, Singapore, 2006.
[6] Z. J. JIANG, *On Stević-Sharma operator from the Zygmund space to the Bloch-Orlicz space*, Adv. Differ. Equ. Vol. 2015, Article no. 228, (2015), 12 pages.
[7] H. LI AND Z. GUO, *On a product-type operator from Zygmund-type spaces to Bloch-Orlicz spaces*, J. Inequal. Appl. Vol. 2015, Article No. 132, (2015), 18 pages.
[8] H. LI AND Z. GUO, *Note on a Li-Stević integral-type operator from mixed-norm spaces to nth weighted spaces*, J. Math. Inequal. **11** (1) (2017), 77–85.
[9] H. LI AND S. LI, *Norm of an integral operator on some analytic function spaces on the unit disk*, J. Math. Inequal. Vol. 2013, Article No. 342, (2013).
[10] S. LI AND S. STEVIĆ, *Integral type operators from mixed-norm spaces to α-Bloch spaces*, Integral Transforms Spec. Funct. **18** (7) (2007), 485–493.
[11] S. LI AND S. STEVIĆ, *Riemann-Stieltjes operators on Hardy spaces in the unit ball of $\mathbb{C}^n$*, Bull. Belg. Math. Soc. Simon Stevin **14** (4) (2007), 621–628.

Paper JMI-19-39

[12] X. LIU AND S. LI, *Norm and essential norm of a weighted composition operator on the Bloch space*, Integral Equations Operator Theory **87** (3) (2017), 309–325.
[13] X. LIU AND S. LI, *Differences of generalized weighted composition operators from the Bloch space into Bers-type spaces*, Filomat **31** (6) (2017), 1671–1680.
[14] Y. LIU AND Y. YU, *On a Stević-Sharma operator from Hardy spaces to the logarithmic Bloch spaces*, J. Inequal. Appl. Vol. 2015, Article No. 22, (2015), 19 pages.
[15] L. MA AND Z. YANG, *Stević-Sharma type operators on Fock spaces in several variables*, Czechoslovak Math. J. **74** (4) (2024), 1241–1263.
[16] S. OHNO, *Products of composition and differentiation on Bloch spaces*, Bull. Korean Math. Soc. **46** (6) (2009), 1135–1140.
[17] A. PELCZYNSKI, *Norms of classical operators in function spaces*, Astérisque **131** (1985), 137–162.
[18] C. POMMERENKE, *Schlichte funktionen und analytische funktionen von beschränkter mittlerer oszillation*, Comment. Math. Helv. **52** (1977), 591–602.
[19] W. RUDIN, *Real and Complex Analysis*, Higher Mathematics Series, 3rd edition, McGraw-Hill Education, London, New York, Sidney, 1976.
[20] W. RUDIN, *Function Theory in the Unit Ball of* $\mathbb{C}^n$, Grundlehren der Mathematischen Wissenschaften, New York, Berlin, Springer-Verlag, 1980.
[21] B. SEHBA AND S. STEVIĆ, *On some product-type operators from Hardy-Orlicz and Bergman-Orlicz spaces to weighted-type spaces*, Appl. Math. Comput. **233** (2014), 565–581.
[22] A. K. SHARMA, *Products of multiplication, composition and differentiation between weighted Bergman-Nevanlinna and Bloch-type spaces*, Turk. J. Math. **35** (2011), 275–291.
[23] A. K. SHARMA, *Weighted composition operators from Cauchy integral transforms to logarithmic weighted-type spaces*, Ann. Funct. Anal. **4** (1) (2013), 163–174.
[24] S. STEVIĆ, *Boundedness and compactness of an integral operator on a weighted space on the polydisc*, Indian J. Pure Appl. Math. **37** (6) (2006), 343–355.
[25] S. STEVIĆ, *Norms of some operators from Bergman spaces to weighted and Bloch-type space*, Util. Math. **76** (2008), 59–64.
[26] S. STEVIĆ, *Composition operators from the weighted Bergman space to the* n*-th weighted spaces on the unit disc*, Discrete Dyn. Nat. Soc. Vol. 2009, Article ID 742019, (2009), 11 pages.
[27] S. STEVIĆ, *Norm of weighted composition operators from* α*-Bloch spaces to weighted-type spaces*, Appl. Math. Comput. **215** (2009), 818–820.
[28] S. STEVIĆ, *Composition followed by differentiation from* H^∞ *and the Bloch space to* n*th weighted-type spaces on the unit disk*, Appl. Math. Comput. **216** (2010), 3450–3458.
[29] S. STEVIĆ, *Weighted differentiation composition operators from the mixed-norm space to the* n*th weigthed-type space on the unit disk*, Abstr. Appl. Anal. Vol. 2010, Article ID 246287, (2010), 15 pages.
[30] S. STEVIĆ, *On a new product-type operator on the unit ball*, J. Math. Inequal. **16** (4) (2022), 1675–1692.
[31] S. STEVIĆ, *Norm of the general polynomial differentiation composition operator from the space of Cauchy transforms to the* m*th weighted-type space on the unit disk*, Math. Methods Appl. Sci. **47** (2024), 3893–3902.
[32] S. STEVIĆ, C. S. HUANG AND Z. J. JIANG, *Sum of some product-type operators from Hardy spaces to weighted-type spaces on the unit ball*, Math. Methods Appl. Sci. **45** (17) (2022), 11581–11600.
[33] S. STEVIĆ AND A. K. SHARMA, *Composition operators from the space of Cauchy transforms to Bloch and the little Bloch-type spaces on the unit disk*, Appl. Math. Comput. **217** (2011), 10187–10194.
[34] S. STEVIĆ AND A. K. SHARMA, *On a product-type operator between Hardy and* α*-Bloch spaces of the upper half-plane*, J. Inequal. Appl. Vol. 2018, Article No. 273, (2018), 18 pages.
[35] S. STEVIĆ, A. K. SHARMA AND A. BHAT, *Products of multiplication composition and differentiation operators on weighted Bergman spaces*, Appl. Math. Comput. **217** (2011), 8115–8125.
[36] S. STEVIĆ, A. K. SHARMA AND R. KRISHAN, *Boundedness and compactness of a new product-type operator from a general space to Bloch-type spaces*, J. Inequal. Appl. Vol. 2016, Article No. 219, (2016), 32 pages.
[37] V. A. TRENOGIN, B. M. PISAREVSKIY AND T. S. SOBOLEVA, *Zadachi i Uprazhneniya po Funktsional'nomu Analizu*, Nauka, Moskva, 1984, (in Russian).

[38] W. YANG, *Generalized weighted composition operators from the $F(p,q,s)$ space to the Bloch-type space*, Appl. Math. Comput. **218** (2012), 4967–4972.
[39] W. YANG AND X. ZHU, *Generalized weighted composition operators from area Nevanlinna spaces to Bloch-type spaces*, Taiwan. J. Math. **16** (3) (2012), 869–883.
[40] F. ZHANG AND Y. LIU, *On the compactness of the Stević-Sharma operator on the logarithmic Bloch spaces*, Math. Inequal. Appl. **19** (2) (2016), 625–642.
[41] X. ZHU, *Generalized weighted composition operators on weighted Bergman spaces*, Numer. Funct. Anal. Opt. **30** (2009), 881–893.
[42] X. ZHU, *Essential norm and compactness of the product of differentiation and composition operators on Bloch type spaces*, Math. Inequal. Appl. **19** (1) (2016), 325–334.
[43] X. ZHU, *Generalized weighted composition operators on weighted Bergman spaces, II*, Math. Inequal. Appl. **22** (2019), 1055–1066.
[44] X. ZHU, *Weighted composition operators from Dirichlet-type spaces into Stević-type spaces*, Math. Inequal. Appl. **23** (4) (2020), 1311–1323.
[45] X. ZHU, J. DU, *Weighted composition operators from weighted Bergman spaces to Stević-type spaces*, Math. Inequal. Appl. **22** (1) (2019), 361-376.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com