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GENERALIZED HARDY'S INEQUALITIES WITH NONLINEAR INTEGRATION LIMITS

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Abstract. In this paper, we establish two integral inequalities arising in the study of weighted norm estimates. First, we consider a sequence of measurable sets forming a partition of $\mathbb{R}^m$ and derive an upper bound for a sum involving weighted integrals of a function g, controlled by a sequence of positive numbers. Second, we prove an integral inequality involving a non-decreasing function h with $\sup(h(x)/x) < \infty$ and a power-weighted integral of f. Higher dimensional analogue of this inequality are also established.

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