Journal of Mathematical Inequalities
Volume 19, Number 3 (2025), 751–763
doi:10.7153/jmi-2025-19-48

A MULTIDIMENSIONAL HARDY–HILBERT'S INTEGRAL INEQUALITY INVOLVING ONE DERIVATIVE FUNCTION OF HIGHER–ORDER

ZHILAI YAN, BICHENG YANG* AND DONGMEI XIN

Abstract. By means of the weight functions, the idea of introduced parameters and using the techniques of real analysis, a multidimensional Hardy-Hilbert's integral inequality involving one derivative function of higher-order is obtained. As applications, the equivalent statements of the best possible constant factor in the new inequality related to several parameters are considered. Some corollaries are obtained.

Mathematics subject classification (2020): 26D15.
Keywords and phrases: Weight function, derivative function of higher-order, parameter, beta function, multidimensional Hardy-Hilbert's integral inequality.

REFERENCES

[1] V. ADIYASUREN, T. BATBOLD AND E. A. AZAR, *A new discrete Hilbert-type inequality involving partial sums*, Journal of Inequalities and Applications, (2019) **2019**: 127.

[2] V. ADIYASUREN, T. BATBOLD AND M. KRNIĆ, *Hilbert-type inequalities involving differential operators, the best constants and applications*, Math. Inequal. Appl. **18** (2015), 111–124.

[3] L. E. AZAR, *The connection between Hilbert and Hardy inequalities*, Journal of Inequalities and Applications (2013) **2013**: 452.

[4] Q. CHEN, B. HE, Y. HONG AND Z. LI, *Equivalent parameter conditions for the validity of half-discrete Hilbert-type multiple integral inequality with generalized homogeneous kernel*, J. Funct. Spaces, (2020), **2020**, 7414861.

[5] G. H. HARDY, J. E. LITTLEWOOD AND G. POLYA, *Inequalities*, Cambridge University Press, Cambridge, 1934.

[6] B. HE, *A multiple Hilbert-type discrete inequality with a new kernel and best possible constant factor*, Journal of Mathematical Analysis and Applications **431** (2015), 902–990.

[7] B. HE, Y. HONG AND Z. LI, *Conditions for the validity of a class of optimal Hilbert type multiple integral inequalities with non-homogeneous*, Journal of Inequalities and Applications, (2021), **2021**: 64.

[8] Y. HONG, *On the structure character of Hilbert's type integral inequality with homogeneous kernel and applications*, J. Jilin. Univ. Sci. Ed. **55** (2), (2017), 189–194.

[9] Y. HONG, Q. L. HUANG, B. C. YANG AND J. Q. LIAO, *The necessary and sufficient conditions for the existence of a kind of Hilbert-type multiple integral inequality with the non-homogeneous kernel and its applications*, Journal of Inequalities and Applications, (2017), **2017**: 316.

[10] Y. HONG AND Y. M. WEN, *A necessary and sufficient condition of that Hilbert type series inequality with homogeneous kernel has the best constant factor*, Ann. Math. **37A** (3), (2016), 329–336.

[11] Y. HONG, Y. R. ZHONG AND B. C. YANG, *A more accurate half-discrete multidimensional Hilbert-type inequality involving one multiple upper limit function*, Axioms, (2023), **12**, 211, https://doi.org/10.3390/axioms12020211.

Paper JMI-19-48

[12] Y. HONG, Y. R. ZHONG AND B. C. YANG, *A more accurate half-discrete multidimensional Hilbert-type inequality involving one multiple upper limit function*, Journal of inequalities and Applications, (2023), **2023**: 74.

[13] Y. HONG, Y. R. ZHONG AND B. C. YANG, *On a more accurate half-discrete multidimensional Hilbert-type inequality involving one derivative function of m-order*, Journal of Inequalities and Applications, (2023), **2023**: 74.

[14] Q. L. HUANG, *A new extension of Hardy-Hilbert-type inequality*, Journal of Inequalities and Applications (2015), **2015**: 397.

[15] M. KRNIĆ AND J. PEČARIĆ, *Extension of Hilbert's inequality*, J. Math. Anal. Appl. **324** (1) (2006), 150–160.

[16] M. KRNIĆ AND J. PEČARIĆ, *General Hilbert's and Hardy's inequalities*, Mathematical Inequalities & Applications **8** (1) (2005), 29–51.

[17] J. C. KUANG, *Applied Inequalitie*, Shandong Science and Technology Press, Jinan, 2004.

[18] J. C. KUANG, *Real and Functional Analysis (Continuation)*, vol. 2, Higher Education Press, Beijing, 2015.

[19] J. Q. LIAO AND B. C. YANG, *A new reverse half-discrete Hilbert-type inequality with one partial sum involving one derivative function of higher order*, Open Mathematics, (2023), **21**: 20230139.

[20] R. C. LUO, B. C. YANG AND L. P. HE, *A Hardy-Hilbert-type integral inequality involving two multiple upper-limit functions*, Journal of Inequalities and Applications, (2023), **2023**: 19.

[21] H. M. MO AND B. C. YANG, *On a new Hilbert-type integral involving the upper limit functions*, Journal of Inequalities and Applications, (2020), **2020**: 5.

[22] I. PERIĆ AND P. VUKOVIĆ, *Multiple Hilbert's type inequalities with a homogeneous kernel*, Banach Journal of Mathematical Analysis **5** (2) (2011), 33–43.

[23] A. Z. WANG AND B. C. YANG, *A new half-discrete Hilbert-type inequality involving one multiple upper limit function and one partial sums*, Annals Mathematics **45A** (1) (2024), 25–38.

[24] Z. T. XIE, Z. ZENG AND Y. F. SUN, *A new Hilbert-type inequality with the homogeneous kernel of degree* -2, Advances and Applications in Mathematical Sciences **12** (7) (2013), 391–401.

[25] D. M. XIN, *A Hilbert-type integral inequality with the homogeneous kernel of zero degree*, Mathematical Theory and Applications **30** (2) (2010), 70–74.

[26] J. S. XU, *Hardy-Hilbert's inequalities with two parameters*, Adv. Math. **36** (2) (2007), 63–76.

[27] B. C. YANG, *The Norm of Operator and Hilbert-type Inequality*, Science Press, Beijing, 2009.

[28] Z. ZENG, K. RAJA RAMA GANDHI AND Z. T. XIE, *A new Hilbert-type inequality with the homogeneous kernel of degree* -2 *and with the integral*, Bulletin of Mathematical Sciences and Applications **3** (1) (2014), 11–20.

[29] Z. H. ZENG AND B. C. YANG, *A Hardy-Hilbert-type integral inequality involving the derivative functions of n-order*, Journal of South China Normal University (Natural Science Editon) **56** (4) (2024), 123–128.