Journal of Mathematical Inequalities
Volume 19, Number 3 (2025), 765–778
doi:10.7153/jmi-2025-19-49

HERMITE–HADAMARD TYPE INTEGRAL INEQUALITIES INVOLVING FRACTIONAL INTEGRALS FOR THE s–CONVEX FUNCTIONS IN THE FOURTH SENSE

GÜLTEKIN TINAZTEPE, ZEYNEP EKEN, SERAP KEMALI, SINEM SEZER EVCAN AND SEVDA SEZER*

Abstract. In this paper, the Hermite-Hadamard type inequalities involving Riemann-Liouville fractional integrals for s-convex functions in the fourth sense are obtained. By means of certain integral identities for fractional integrals, some inequality relations among fractional integrals of the fourth sense s-convex functions and the functions whose derivatives are s-convex in the fourth sense are set forth. Moreover, based on the obtained results, some inequality relations among special functions including beta, incomplete beta functions, and means including logarithmic and arithmetic mean are given as applications.

Mathematics subject classification (2020): 26A33, 26D07, 26D10, 26D15, 26A51.
Keywords and phrases: s-convex function in the fourth sense, Hermite-Hadamard type inequality, Riemann-Liouville fractional integrals, beta function.

REFERENCES

[1] G. R. ADILOV, S. KEMALI, *Abstract convexity and Hermite-Hadamard type inequalities*, Journal of Inequalities and Applications, **2009**, 1–13, 2009.
[2] G. M. BOHNER, A. KASHURI, P. O. MOHAMMED, VALDES JEN, *Hermite-Hadamard-type Inequalities for Conformable Integrals*, Hacettepe Journal of Mathematics and Statistics, **51** (3), 775–786, 2022.
[3] G. W. W. BRECKNER, *Stetigkeitsaussagen für eine Klasse verallgemeinerter konvexer Funktionen in topologischen linearen Räumen*, Publ. Inst. Math. (Beograd) (NS), **23**, 37, 13–20, 1978.
[4] G. Z. DAHMANI, *On Minkowski and Hermite-Hadamard Integral Inequalities via Fractional Integration*, Annals of Functional Analysis, **1**, 51–58, 2010.
[5] G. S. S. DRAGOMIR, CEM PEARCE, *Selected Topics on Hermite-Hadamard Inequalities and Applications*, RGMIA Monographs, Victoria University, 2000.
[6] Z. EKEN, *Hermite-Hadamard Type Inequalities Related to s-Convex Functions in the Third Sense*, Turkish Journal of Mathematics and Computer Science, **13** (2), 318–330, 2021.
[7] Z. EKEN, S. KEMALI, G. TINAZTEPE, G. ADILOV, *The Hermite-Hadamard inequalities for p-convex functions*, Hacettepe Journal of Mathematics and Statistics, **50** (5), 1268–1279, 2021.
[8] Z. EKEN, S. SEZER, G. TINAZTEPE, G. ADILOV, *s-Convex functions in the fourth sense and some of their properties*, Konuralp Journal of Mathematics, **9** (2), 260–267, 2021.
[9] Z. EKEN, *Fejer inequality for s-convex functions in the fourth sense*, Journal of mathematical inequalities **16** (1), 19–34, 2022.
[10] A. FERNANDEZ, P. MOHAMMED, *Hermite Hadamard inequalities in fractional calculus defined using Mittag Leffler kernels*, Mathematical Methods in the Applied Sciences, **44** (10), 8414–8431, 2021.
[11] M. JLELI, D. O'REGAN, B. SAMET, *On Hermite-Hadamard type inequalities via generalized fractional integrals*, Turkish Journal of Mathematics, **40** (6), 1221–1230, 2016.
[12] U. N. KATUGAMPOLA, *A new approach to generalized fractional derivatives*, Bull. Math. Anal. App., **6**, 1–15, 2014.

Paper JMI-19-49

[13] S. KEMALI, *Hermite-Hadamard Type Inequality for s-Convex Functions in the Fourth Sense*, Turkish Journal of Mathematics and Computer Science, **13** (2), 287–293, 2021.
[14] S. KEMALI, S. SEZER, G. TINAZTEPE, G. ADILOV, *s-Convex functions in the third sense*, Korean Journal of Mathematics, **29** (3), 593–602, 2021.
[15] A. A. KILBAS, O. I. MARICHEV, S. G. SAMKO, *Fractional Integrals and Derivatives*, Theory and Applications, Gordon and Breach, Switzerland, 1993.
[16] Y. C. KWUN, M. TANVEER, W. NAZEER, K. GDAWIEC AND S. M. KANG, *Mandelbrot and Julia Sets via Jungck-CR Iteration With s-Convexity*, in IEEE Access, vol. 7, pp. 12167–12176, 2019.
[17] Z. LIN, J. R. WANG, *New Riemann-Liouville fractional Hermite-Hadamard inequalities via two kinds of convex functions*, Journal of Interdisciplinary Mathematics, **20** (2), 357–382, 2017.
[18] P. O. MOHAMMED, *Hermite Hadamard inequalities for Riemann Liouville fractional integrals of a convex function with respect to a monotone function*, Math. Meth. Appl. Sci., **44**, 2314–2324, 2021.
[19] W. ORLICZ, *A note on modular spaces*, I, Bull. Acad. Polon. Sci. Ser. Sci. Math., Astr. et Phys., **9**, 157–162, 1961.
[20] M. Z. SARIKAYA, E. SET, H. YALDIZ, N. BAŞAK, *Hermite-Hadamard's inequalities for fractional integrals and related fractional inequalities*, Mathematical and Computer Modelling, **57** (9–10), 2403–2407, 2013.
[21] E. SET, M. Z. SARIKAYA, M. E. OZDEMIR, H. YILDIRIM, *The Hermite-Hadamard's inequality for some convex functions via fractional integrals and related results*, Journal of Applied Mathematics, Statistics and Informatics, **10** (2), 69–83, 2014.
[22] S. SEZER, *Hermite-Hadamard Type Inequalities for the Functions Whose Absolute Values of First Derivatives are p-Convex*, Fundamental Journal of Mathematics and Applications, **4** (2), 88–99, 2021.
[23] S. SEZER, *The Hermite-Hadamard inequality for s-Convex functions in the third sense*, AIMS Mathematics, **6** (7), 7719–7732, 2021.
[24] S. SEZER, Z. EKEN, G. TINAZTEPE, G. ADILOV, *p-convex Functions and Some of Their Properties*, Numerical Functional Analysis and Optimization, **42** (4), 443–459, 2021.