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DIAGONAL–SCHUR COMPLEMENTS OF DASHNIC–ZUSMANOVICH TYPE MATRICES

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Abstract. The subclasses of nonsingular H-matrices are very important in the field of numerical algebra and related fields. As a subclass of H-matrices, the class of Dashnic-Zusmanovich type matrices was first mentioned in 2018. This paper investigates the closure properties of diagonal-Schur complements on DZT matrices. We prove that the diagonal-Schur complements of DZT matrices with respect to any index set are still in the same matrix class, which improves the corresponding result obtained by Li, Huang, and Zhao in 2022 (Chaoqian Li, Zhengyu Huang and Jianxing Zhao, Linear and Multilinear Algebra, 70 (2022): 4071–4096). Numerical examples are given to verify the correctness of the proposed results.

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