Journal of Mathematical Inequalities
Volume 19, Number 3 (2025), 893–907
doi:10.7153/jmi-2025-19-57

BEREZIN RADIUS INEQUALITIES VIA CLASSICAL INEQUALITIES

SUNA SALTAN, MEHMET GÜRDAL, MUBARIZ T. GARAYEV* AND RUCHI SINGH RAJAWAT

Abstract. We use classical inequalities, including the Cauchy-Schwarz inequality and its extension, the Buzano inequality, as well as their generalizations, to prove new Berezin radius inequalities for operators on reproducing kernel Hilbert spaces.

Mathematics subject classification (2020): 47A30, 47A63.
Keywords and phrases: Cauchy-Schwarz inequality, reproducing kernel, Berezin radius, Berezin norm.

REFERENCES

[1] M. W. ALOMARI, M. HAJMOHAMADI AND M. BAKHERAD, *Norm-parallelism of Hilbert space operators and the Davis-Wielandt Berezin number*, J. Math. Inequal., **17** (2023), 231–258.

[2] N. ARONZAJN, *Theory of reproducing kernels*, Trans. Amer. Math. Soc., **68** (1950), 337–404.

[3] M. BAKHERAD, M. HAJMOHAMADI, R. LASHKARIPOUR AND S. SAHOO, *Some extensions of Berezin number inequalities on operators*, Rocky Mountain J. Math., **51** (2021), 1941–1951.

[4] H. BAŞARAN, M. GÜRDAL AND A. N. GÜNCAN, *Some operator inequalities associated with Kantorovich and Hölder-McCarthy inequalities and their applications*, Turkish J. Math., **43** (2019), 523–532.

[5] P. BHUNIA, S. JANA, M. S. MOSLEHIAN AND K. PAUL, *Improved inequalities for numerical radius via cartesian decomposition*, Funktsional. Anal. I Prilozhen, **57** (2023), 24–37.

[6] P. BHUNIA, M. T. GARAYEV, K. PAUL AND R. TAPDIGOGLU, *Some new applications of Berezin symbols*, Complex Analy. Oper. Theory, **17** (2023), 96.

[7] P. BHUNIA, M. GÜRDAL, K. PAUL, A. SEN AND R. TAPDIGOGLU, *On a new norm on the space of reproducing kernel Hilbert space operators and Berezin radius inequalities*, Numer. Funct. Anal. Optim., **44** (2023), 970–986.

[8] M. L. BUZANO, *Generalizzazione della diseguaglianza di Cauchy-Schwarz*, Rend. Sem. Mat. Univ, e Politec. Torino, **31** (1974), 405–409.

[9] T. BOTTAZI AND C. CONDE, *Generalized Buzano inequality*, arXiv: 2204.14233.

[10] F. CHIENA, E. F. MOHOMMED, M. HAJMOHAMADIC AND R. LASHKARIPOUR, *Inequalities of Generalized Euclidean Berezin Number*, Filomat, **36** (2022), 5337–5345.

[11] S. S. DRAGOMIR, *A survey on Cauchy-Bunyakovsky-Schwarz type discrete inequalities*, JIPAM, J. Inequal. Pure Appl. Math., **4** (2003), 63, 142 pp.

[12] S. S. DRAGOMIR, *Power inequalities for the numerical radius of a product of two operators in Hilbert spaces*, Sarajevo J. Math., **5** (2009), 269–278.

[13] M. ENGLIŠ, *Toeplitz operators and the Berezin transform on H^2*, Lin. Alg. Appl., **223/224** (1995), 171–204.

[14] M. HAJMOHAMADI, R. LASHKARIPOUR AND M. BAKHERAD, *Improvements of Berezin number inequalities*, Linear Multilinear Algebra, **68** (2020), 1218–1229.

[15] P. HALMOS, *A Hilbert Space Problem Book*, Springer, Berlin, (1982).

Paper JMI-19-57

[16] M. HOSSEINI, R. NURAEI AND M. S. HOSSEINI, *Generalized Cauchy-Bunyakovsky-Schnarz inequalities*, Funct, Anal. Its Appl., **568** (2024), 106–120.

[17] M. T. GARAYEV, M. GÜRDAL AND S. SALTAN, *Hardy type inequality for reproducing kernel Hilbert space operators and related problems*, Positivity, **21** (2017), 1615–1623.

[18] M. GARAYEV, H. GUEDIRI AND F. ALSHORMANI, *Upper bounds for Berezin numbers of self-adjoint operators and applications*, Baku Mathematical Journal, **3** (2024), 24–34.

[19] M. T. GARAYEV, H. GUEDIRI, M. GÜRDAL AND G. M. ALSAHLI, *On some problems for operators on the reproducing kernel Hilbert space*, Linear Multilinear Algebra, **69** (2021), 2059–2077.

[20] M. GUESBA AND M. GARAYEV, *Estimates for the Berezin number inequalities*, J. Pseudo-Differ. Oper. Appl., **15** (2024), 43.

[21] M. GÜRDAL AND R. TAPDIGOGLU, *New Berezin radius upper bounds*, Proc. Ins. Math. Mech., **45** (2023), 210–218.

[22] M. GÜRDAL, G. ERKAN AND M. GARAYEV, *Berezin norm and Berezin radius inequalities of product and sums with Selberg inequality*, Proc. Ins. Math. Mech., **50** (2024), 258–273.

[23] M. T. KARAEV, *Berezin symbol and invertibility of operators on the functional Hilbert spaces*, J. Funct. Anal., **238** (2006), 181–192.

[24] M. T. KARAEV, *Reproducing kernels and Berezin symbols techniques invarious questions of operator theory*, Complex Anal. Oper. Theory, **7** (2013), 983–1018.

[25] M. T. KARAEV, *Berezin symbols and Schatten-von Neumann classes*, Math. Notes, **2** (2002), 185–192.

[26] M. T. KARAEV, *Berezin set and Berezin number of operators and their applications*, The 8 th Workshop on Numerical Ranges and Numerical Radii (WONRA-O6), University of Bremen, 2006, pp. 14.

[27] M. T. KARAEV, *On the Riccati equations*, Monatsh. für Math., **155** (2008), 161–166.

[28] M. T. KARAEV, M. GÜRDAL AND M. B. HUBAN, *Reproducing kernels, Englis algebras and some applications*, Studia Math., **232** (2016), 113–141.

[29] M. KARAEV, M. GÜRDAL AND S. SALTAN, *Some applications of Banach algebra techniques*, Math. Nachr., **284** (2011), 1678–1689.

[30] T. KATO, *Notes on some inequalities for linear operators*, Math. Ann., **125** (1952), 208–212.

[31] F. KITTANEH AND H. R. MORADI, *Cauchy-Schwarz type inequalities and applications to numerical radius inequalities*, Math. Ineq. Appl., **23** (2020), 1117–1125.

[32] F. KITTANEH, *Notes on some inequalities for Hilbert space operators*, Pabl. Res. Inst. Math. Sci., **24** (1988), 283–293.

[33] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Stud. Math., **158** (2003), 11–17.

[34] F. KITTANEH, *Bounds for the zeros of polynomials from matrix inequalities*, Arch. Math. (Basel), **81** (2003), 601–608.

[35] M. KHOSRAVI, R. DRNOVŠEK AND M. S. MOSLEHIAN, *A commutator approach to Buzano's inequality*, Filomat, **26** (2012), 827–832.

[36] C. P. NICULESCU AND L.-E. PERSSON, *Convex functions and their applications. A contemporary approach*, CMS Books Math., Ourrages Math. SMC, **23**, Springer, 2006, 255 pp.

[37] V. I. PAULSEN AND M. RAGHUPATHI, *An introduction to the theory of reproducing kernel Hilbert spaces*, Cambridge Studies in Advanced Mathematics, vol. 152, Cambridge Univ Press, Cambridge, 2016, MR 3526117.

[38] T. PEČARIC, J. M. FURUTA, Y. HOT, Y. SEO, *Mond-Pečaric method in operator inequalities. Inequalities for bounded selfadjoint operators on a Hilbert space*, Monogr. Inequal., vol. 1, Element, Zagreb, 2005, xiv+265 pp.

[39] S. SALTAN, R. TAPDIGOGLU AND I. ÇALIŞIR, *Some new relations between the Berezin number and the Berezin norm of operators*, Rocky M. J. Math., **52** (2022), 1767–1774.

[40] R. TAPDIGOGLU, *New Berezin symbol inequalities for operators on the reproducing kernel Hilbert space*, Oper. Matrices, **15** (2021), 1031–1043.

[41] R. TAPDIGOGLU AND M. GARAYEV, *Generalized Riccati operator equations, invariant subspaces and Toeplitz Corona problem*, Numer. Funct. Anal. Optim., **46** (2025), 262–274.

[42] R. TAPDIGOGLU, M. GARAYEV AND N. ALTWAIJRY, *Berezin symbol inequalities via Grüss type inequalities and related questions*, Turkish J. Math., **46** (2022), 991–1003.

[43] R. TAPDIGOGLU, M. GÜRDAL, N. ALTWAIJRY AND N. SARI, *Davis-Wielandt-Berezin radius inequalities via Dragomir inequalities*, Oper. Matrices, **15** (2021), 1445–1460.

[44] U. YAMANCI, R. TUNÇ AND M. GÜRDAL, *Berezin number, Grüss-type inequalities and their applications*, Bull. Malaysian Math. Sci. Soc., **43** (2020), 2287–2296.

[45] K. ZHU, *Operator theory in function spaces*, sec. ed., Math. Surveys and Monographs, vol. 138, Amer. Math. Soc., Providence, RI, 2007, MR 2311536.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com