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TIGHTENING BOUNDS ON THE NUMERICAL RADIUS FOR HILBERT SPACE OPERATORS

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Abstract. Let S be a bounded linear operator on a Hilbert space. We show that if S is accretive (resp. dissipative in the sense that $\frac{S-S^*}{2i}$ is positive) in the sense that $\frac{S+S^*}{2}$ is positive, then

$$\frac{1}{2}\sqrt{\|S\|^2+\|\Re S\|\,\|\Im S\|}\leqslant \omega(S),$$

where $\|\cdot\|$ and $\omega(\cdot)$ denote the operator norm and the numerical radius, respectively.

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