Journal of
Mathematical
Inequalities
Volume 19, Number 3 (2025), 953–965 doi:10.7153/jmi-2025-19-62

BILINEAR FRACTIONAL INTEGRAL OPERATORS ON GRAND MORREY SPACES AND GRAND HARDY–MORREY SPACES

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Abstract. We establish the boundedness of bilinear fractional integral operators on grand Morrey spaces and grand Hardy-Morrey spaces. Our approach combines a refined extrapolation method with sparse domination techniques, extending recent results on linear fractional integrals to the multilinear setting. The key innovation is the adaptation of the extrapolation machinery to handle the nonlinear nature of bilinear operators while preserving the delicate balance between the fractional parameter and the integrability indices. As applications, we obtain bilinear Sobolev embeddings and fractional Leibniz rules in the grand Morrey space framework.

Mathematics subject classification (2020): Primary 42B20; Secondary 42B25, 42B35, 46E30, 26A33.
Keywords and phrases: Bilinear fractional integral, grand Morrey space, grand Hardy-Morrey space, extrapolation, weighted inequality, sparse operator.

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