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THE WEIGHTED POWER DIFFERENCE MEAN AND ITS GENERALIZATION

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Abstract. Pal, Singh, Moslehian and Aujla obtained the inequalities for a convex function and introduced the weighted logarithmic mean for two positive numbers or bounded linear operators on a complex Hilbert space. Furuichi and Minculete refined the inequalities by Pal et al.

In this paper, based on their results, we newly introduce the weighted power difference mean as a generaliation of the weighted logarithmic mean. We show relations among the weighted power, power difference and arithmetic means. Moreover, we obtain its generalization by considering the notion of a transpose symmetric path of t-weighted operator means.

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