COMPLETE MOMENT CONVERGENCE FOR ρ^*–MIXING LINEAR PROCESSES WITH RANDOM COEFFICIENTS AND ITS APPLICATIONS

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Abstract. In this paper, we will study the complete moment convergence for the dependent linear processes under some suitable conditions, which $Y_t = \sum_{j=-\infty}^{\infty} A_j X_{t-j}$ be a dependent linear process, where the $\{X_n, n \in \mathbb{Z}\}$ is a sequence of ρ^*-mixing random variables, with stochastically dominated a random variable X, and $\{A_n, n \in \mathbb{Z}\}$ is a sequence independent random variables. As applications, we will present Marcinkiewicz-Zygumund strong laws and strong laws of large numbers for this linear processes. Finally, we also present some numerical simulations to demonstrate the finite sample performances of the theoretical results.

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