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COMPLETE PICTURE ON THE MONOTONICITY CHARACTER OF A CLASS OF SEQUENCES WITH A PARAMETER

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Abstract. We describe the monotonicity character of the class of sequences $z_n^{(\alpha)} := (1-\alpha)x_n + \alpha y_n$, $n \in \mathbb{N}$, where the parameter α belongs to the interval $[0,1]$, $x_n = \sum_{j=1}^{n} \frac{1}{\sqrt{j}} - 2\sqrt{n}$, and $y_n = \sum_{j=1}^{n} \frac{1}{\sqrt{j}} - 2\sqrt{n+1}$, on the whole domain, that is, on the set $\mathbb{N}$. If for some value of the parameter α the sequence is not strictly decreasing or strictly increasing on the whole domain, we determine the exact value of the index n where the monotonicity is changed, as well as the types of the monotonicity before and after the value of the index. A comparison of the problem of describing the monotonicity character of the sequence on the whole domain and the problem of describing its eventual monotonicity, as well as some methods for dealing with the problems, is also given.

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