

NEW WEIGHT CHARACTERIZATIONS FOR THE DISCRETE HARDY INEQUALITY WITH KERNEL

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Abstract. In this work, we present new pairs of equivalent conditions for the validity of discrete weighted Hardy inequality with kernel satisfying Oinarov's conditions for the parameters $1 < p \leq q < \infty$. In addition, we give lower and upper estimates for the best constant of the inequality.

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