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ANALYSIS OF MACLAURIN'S INEQUALITY WITH APPLICATIONS IN NUMERICAL ANALYSIS

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Abstract. Maclaurin's inequality estimates the error bounds of a three-point open method named as Maclaurin's procedure. The current study aims to explore the error boundaries of Maclaurin's rule by utilizing the convexity of the mappings. We derive a new twice-differentiable Maclaurin's identity. Based on newly developed identity, the convexity of mappings, and the elementary properties of inequalities, we derive some new Maclaurin's type inequalities. Also, we apply the obtained bounds to formulate the relation between means, composite quadrature bounds, and a novel two-step iterative method with a cubic order of convergence. Lastly, we explore our major findings and the iterative method through illustrative examples and visuals.

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