

## HÖLDER TYPE INEQUALITIES FOR MATRICES

TSUYOSHI ANDO AND FUMIO HIAI

*Abstract.* We discuss Hölder type inequalities involving  $(A^p + B^p)^{1/p}$  for positive semi-definite matrices  $A, B$ . Matrix or trace inequalities of Hölder type as well as weak majorizations of similar type are obtained. Also we give counter-examples for expected Hölder type inequalities.

*Mathematics subject classification (1991):* Primary 15A45, 15A42; Secondary 15A60, 47A63.

*Key words and phrases:* Positive semi-definite matrices, Hölder inequality, trace inequalities, weak majorization, Oppenheim inequality.

### REFERENCES

- [1] T. ANDO, *Concavity of certain maps on positive matrices and applications to Hadamard products*, Linear Algebra Appl. **26** (1979), 203–241.
- [2] T. ANDO, *Majorizations, doubly stochastic matrices, and comparison of eigenvalues*, Linear Algebra Appl. **118** (1989), 163–248.
- [3] T. ANDO, *Majorizations and inequalities in matrix theory*, Linear Algebra Appl. **199** (1994), 17–67.
- [4] T. ANDO AND F. HIAI, *Log majorization and complementary Golden-Thompson type inequalities*, Linear Algebra Appl. **197/198** (1994), 113–131.
- [5] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New-York, 1997.
- [6] K. FAN, *On a theorem of Weyl concerning eigenvalues of linear transformations. II*, Proc. Nat. Acad. Sci. U.S.A. **36** (1950), 31–35.
- [7] F. HANSEN AND G.K. PEDERSEN, *Jensen's inequality for operators and Löwner's theorem*, Math. Ann. **258** (1982), 229–241.
- [8] F. HIAI AND D. PETZ, *The Golden-Thompson trace inequality is complemented*, Linear Algebra Appl. **181** (1993), 153–185.
- [9] T. KATO, *Spectral order and a matrix limit theorem*, Linear and Multilinear Algebra **8** (1979), 15–19.
- [10] H. KOSAKI, *Some remarks on the positive operator  $(a^2 + b^2)^{1/2}$* , unpublished notes.
- [11] F. KUBO AND T. ANDO, *Means of positive linear operators*, Math. Ann. **246** (1980), 205–224.
- [12] E.H. LIEB, *Convex trace functions and the Wigner-Yanase-Dyson conjecture*, Adv. Math. **11** (1973), 267–288.
- [13] A.W. MARSHALL AND I. OLKIN, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.
- [14] A. OPPENHEIM, *Inequalities connected with definite Hermitian forms, II*, Amer. Math. Monthly **61** (1954), 463–466.
- [15] M. TAKESAKI, *Theory of Operator Algebras I*, Springer-Verlag, New York-Heidelberg-Berlin, 1979.
- [16] G. VISICK, *Majorizations of Hadamard products of matrix powers*, Linear Algebra Appl., to appear.