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# STABLE SEMINORMS REVISITED

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*Abstract.* A seminorm $S$ on an algebra $\mathscr{A}$ is called *stable* if for some constant $\sigma > 0$,

$$S(x^k) \leqslant \sigma S(x)^k \quad \text{for all} \quad x \in \mathscr{A} \quad \text{and all} \quad k = 1, 2, 3, \ldots.$$

We call $S$ *strongly stable* if the above holds with $\sigma = 1$. In this note we use several known and new results to shed light on the concepts of stability. In particular, the interrelation between stability and similar ideas is discussed.

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