

ON EQUIVALENCE OF COEFFICIENT CONDITIONS AND APPLICATION

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Abstract. We show that the simplest coefficient condition

$$\sum_{n=1}^{\infty} |c_n|^q \gamma_n \mu_n < \infty,$$

under specific assumptions on the sequence $\{\mu_n\}$, is equivalent to the conditions

$$\sum_{m=1}^{\infty} \beta_m \left(\sum_{n=1}^m \gamma_n |c_n|^q \right)^{p/q} < \infty,$$

and

$$\sum_{m=1}^{\infty} \lambda_m \left(\sum_{n=m}^{\infty} \gamma_n |c_n|^q \right)^{p/q} < \infty,$$

respectively. Plainly the assumptions on $\{\mu_n\}$ depend on $\{\beta_m\}$, or $\{\lambda_m\}$, and $0 < p < q$.

An application to absolute $|C, \alpha|$ -summability of general orthogonal series is also presented.

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