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WHY HÖLDER'S INEQUALITY SHOULD BE CALLED ROGERS' INEQUALITY

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Abstract. The inequality

$$\sum_{k=1}^{n} a_k b_k \leqslant \left(\sum_{k=1}^{n} a_k^p\right)^{1/p} \left(\sum_{k=1}^{n} b_k^q\right)^{1/q} \tag{1}$$

was proved in slightly different form by Rogers in 1888 and then by Hölder in 1889 (Hölder even refered to Rogers!). Today everybody refer to (1) as the Hölder inequality. We will try to explain the history of this and closely related fundamental inequalities with the answer to the question: why the Rogers inequality is called the Hölder inequality? We claim that the Hölder inequality ought to be referred to as the Rogers inequality or at least as the Rogers-Hölder inequality.

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