

WHY HÖLDER'S INEQUALITY SHOULD BE CALLED ROGERS' INEQUALITY

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Abstract. The inequality

$$\sum_{k=1}^n a_k b_k \leq \left(\sum_{k=1}^n a_k^p \right)^{1/p} \left(\sum_{k=1}^n b_k^q \right)^{1/q} \quad (1)$$

was proved in slightly different form by Rogers in 1888 and then by Hölder in 1889 (Hölder even referred to Rogers!). Today everybody refer to (1) as the Hölder inequality. We will try to explain the history of this and closely related fundamental inequalities with the answer to the question: why the Rogers inequality is called the Hölder inequality? We claim that the Hölder inequality ought to be referred to as the Rogers inequality or at least as the Rogers-Hölder inequality.

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REFERENCES

[An asterisk denotes that I have not seen this article in original form but I have found a discussion in secondary sources such as citation by other authors.]

- [1] E. F. BECKENBACH AND R. BELLMAN, *Inequalities*, Springer-Verlag, 1983.
- [2] P. S. BULLEN, D. S. MITRINOVIC AND P. M. VASIC, *Means and Their Inequalities*, D. Reidel Publishing Company, Dordrecht, 1988.
- [3]* V. Y. BUNYAKOVSKI, *Sur quelques inégalités concernant les intégrales ordinaires et les intégrales aux différences finites*, Mémoires de l'Acad. de St. Pétersbourg (VII)1 (1859), No. 9, 1–18.
- [4]* A. L. CAUCHY, *Cours d'Analyse de l'École Royale Polytechnique*, Ire partie: *Analyse Algébrique*, Paris, Debure frères, 1821 Also in *Oeuvres Complètes d'Augustin Cauchy*, Sér II, T. 3, Gauthier-Villars, Paris 1987.
- [5] A. L. DIXON, *Leonard James Rogers*, J. London Math. Soc. **9** (1934), 237–240.
- [6] U. DUDLEY, *Real Analysis and Probability*, Wadsworth, 1989.
- [7] J. HADAMARD, *Etude sur les propriétés des fonctions entières et en particulier d'une fonction considérée par Riemann*, J. Math Pures Appl. Sér. IV **T. 9** (1983), 171–215.
- [8] G. H. HARDY, *Note on a theorem of Hilbert*, Math. Zeit. **6** (1920), 314–317.
- [9] ———, *Prolegomena to a chapter on inequalities*, J. London Math. Soc. **4** (1929), 61–78 and addenda **5** (1929), 80; Also in *Collected papers of G. H. Hardy*, Vol II, Oxford Univ. Press 1967, 471–489.
- [10] ———, *Ramanujan*, Chelsea Publ. Co., New York, 1940.
- [11] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, Cambridge Univ. Press, 1934.
- [12] E. HELLINGER AND O. TOEPLITZ, *Integralgleichungen und Gleichungen mit Unendlichvielen Unbekannten*, Sonderausgabe der Encyclopädie der Mathematischen Wissenschaften, 1927, Also Chelsea Publishing Company, New York 1953.

- [13] E. HÖLDER, *Otto Ludwig Hölder*, Dictionary of Scientific Biography, Vol. 5, Charles Scribner's Sons, New York, 1981, 472–474.
- [14] O. HÖLDER, *Über einen Mittelwertsatz*, Nachr. Akad. Wiss. Göttingen Math.–Phys. Kl., 1889, pp. 38–47.
- [15] J. L. W. V. JENSEN, *Sur les fonctions convexes et les inégalités entre les valeurs moyennes*, Acta Math. **30** (1906), 175–193.
- [16] H. C. KENNEDY, *Who discovered Boyer's law?*, Amer. Math. Monthly **79** (1972), 66–67.
- [17]* J. L. LAGRANGE, *Nouv. Mém. Acad. Berlin*, 1773, Also in Oeuvres, T. 3, Gauthier–Villars, Paris 1869, p. 662f.
- [18] E. LANDAU, *Über einen Konvergenzsatz*, Göttinger Nachrichten (1907), 25–27, Also in *Edmund Landau Collected Works*, Vol. 3, Thales Verlag, Essen 1985, 273–275.
- [19]* A. M. LYAPUNOV (LIAPUNOFF), *Nouvelle forme du théorème sur la limite de probabilité*, Memoires de l'Acad. de St. Pétersbourg (**VIII**) **12** (1901), No. 5, 1–24.
- [20] H. MINKOWSKI, *Geometrie der Zahlen I*, Leipzig, 1896, pp. 115–117.
- [21] D. S. MITRINOVIC, *Analytic Inequalities*, Springer–Verlag, 1970.
- [22] A. PRINGSHEIM, *Zur theorie der ganzen transzendenten Funktionen*, Sitzungsberichte der Mathematisch-Physikalischen Klasse der Königlich Bayerischen Akademie der Wissenschaften **32** (1902), 163–192.
- [23] F. RIESZ, *Untersuchungen über Systeme integrierbarer Funktionen*, Math. Ann. **69** (1910), 449–497.
- [24] ———, *Les Systèmes D'Équations Linéaires a Une Infinite D'Inconnues*, Gauthier–Villars, Paris, 1913.
- [25] ———, *Su alcune disuguaglianze*, Bollettino dell'Unione Mat. Italiana **7** (1928), 77–79.
- [26] L. J. ROGERS, *An extension of a certain theorem in inequalities*, Messenger of Math. **17** (1888), 145–150.
- [27]* H. A. SCHWARZ, *Über ein die Flächen kleinsten Flächeninhalts betreffendes Problem der Variationsrechnung*, Acta Soc. Scient. Fenn. **15** (1885), 315–362; Also in *Gesammelte Mathematische Abhandlungen*, Band I, Springer, Berlin 1890, 224–269.
- [28] O. STOLZ AND J. A. GMEINER, *Theoretische Arithmetik I*, Teubner, Leipzig, 1900.
- [29] B. SZ.-NAGY, *Introduction to Real Functions and Orthogonal Expansions*, Oxford Univ. Press, New York, 1965.
- [30] B. L. VAN DER WAERDEN, *Nachruf auf Otto Hölder*, Math. Ann. **116** (1939), 157–165.