

INEQUALITIES RELATED TO ISOTONICITY OF PROJECTION AND ANTIPROJECTION OPERATORS

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Abstract. A sharp inequality named “the property of four elements” has recently been proved and studied in [6] and [12]. One particular reason for this is that the inequality is closely related to the isotonicity of the projection operator onto a closed convex set in an ordered Hilbert space.

In this paper we prove and study a dual reversed sharp inequality. Moreover, we introduce the concept of antiprojection operator onto a compact non-empty set of a Hilbert space and prove that our new inequality is closely related to the isotonicity of such an operator. Moreover, we prove that both of these inequalities hold also in the reversed direction but of course with other constants.

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