

FUNDAMENTAL SOLUTIONS TO SOME ELLIPTIC EQUATIONS WITH DISCONTINUOUS SENIOR COEFFICIENTS AND AN INEQUALITY FOR THESE SOLUTIONS

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Abstract. Let $Lu := \nabla \cdot (a(x)\nabla u) = -\delta(x - y)$ in \mathbb{R}^3 , $0 < c_1 \leq a(x) \leq c_2$, $a(x)$ is a piecewise-smooth function with the discontinuity surface S which is smooth. It is proved that in an neighborhood of S the behavior of the function u is given by the formula:

$$u(x, y) = \begin{cases} (4\pi a_+)^{-1} [r_{xy}^{-1} + bR^{-1}], & y_3 > 0, \\ (4\pi a_-)^{-1} [r_{xy}^{-1} - bR^{-1}], & y_3 < 0. \end{cases} \quad (*)$$

Here the local coordinate system is chosen in which the origin lies on S , the plane $x_3 = 0$ is tangent to S , $a_+(a_-)$ is the limiting value of $a(x)$ on S from the half-space $x_3 > 0$, ($x_3 < 0$), $r_{xy} := |x - y|$, $R := \sqrt{\rho^2 + (|x_3| + |y_3|)^2}$, $\rho := \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$, $b := (a_+ - a_-)/(a_+ + a_-)$. If S is the plane $x_3 = 0$ and $a(x) = a_+$ in $x_3 > 0$, $a(x) = a_-$ in $x_3 < 0$, then $(*)$ is the global formula for u in \mathbb{R}^3 . Inequality for the fundamental solution for small and large $|x - y|$ follows from formula $(*)$.

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REFERENCES

- [K] KOZLOV, S., *Asymptotics of the fundamental solutions of differential equations of second order*, Matem. Sbornik **113** no. N2 (1980), 302–323, (Russian).
- [LSW] LITTMAN, W., STAMPALCHIA, G., WEINBERGER, H., *Regular points for elliptic equations with discontinuous coefficients*, Ann. Scuola Norm. Super. Pisa 17 (1963), 43–77.
- [R] RAMM, A. G., *Multidimensional Inverse Scattering Problems*, Longman/Wiley, New York, 1992, pp. 1–496, expanded Russian edition, MIR, Moscow, 1994, pp. 1–496.