FUNDAMENTAL SOLUTIONS TO SOME ELLIPTIC EQUATIONS WITH DISCONTINUOUS SENIOR COEFFICIENTS AND AN INEQUALITY FOR THESE SOLUTIONS

A. G. RAMM

Abstract. Let \( Lu := \nabla \cdot (a(x)\nabla u) = -\delta(x - y) \) in \( \mathbb{R}^3 \), \( 0 < c_1 \leq a(x) \leq c_2 \), \( a(x) \) is a piecewise-smooth function with the discontinuity surface \( S \) which is smooth. It is proved that in an neighborhood of \( S \) the behavior of the function \( u \) is given by the formula:

\[
  u(x, y) = \begin{cases} 
    (4\pi a_+)^{-1}[r_{xy}^{-1} + bR^{-1}], & y_3 > 0, \\
    (4\pi a_-)^{-1}[r_{xy}^{-1} - bR^{-1}], & y_3 < 0. 
  \end{cases} \tag{*}
\]

Here the local coordinate system is chosen in which the origin lies on \( S \), the plane \( x_3 = 0 \) is tangent to \( S \), \( a_+(a_-) \) is the limiting value of \( a(x) \) on \( S \) from the half-space \( x_3 > 0 \), \( x_3 < 0 \), \( r_{xy} := |x - y| \), \( R := \sqrt{\rho^2 + (|x_3| + |y_3|)^2} \), \( \rho := \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \), \( b := (a_+ - a_-)/(a_+ + a_-) \). If \( S \) is the plane \( x_3 = 0 \) and \( a(x) = a_+ \) in \( x_3 > 0 \), \( a(x) = a_- \) in \( x_3 < 0 \), then \((*)\) is the global formula for \( u \) in \( \mathbb{R}^3 \). Inequality for the fundamental solution for small and large \( |x - y| \) follows from formula \((*)\).

Key words and phrases: Fundamental solutions, elliptic equations, discontinuous coefficients, inverse problems.

REFERENCES

