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AN UPPER BOUND FOR THE ZEROS OF THE CYLINDER FUNCTION $C_\nu(x)$

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Abstract. For large values of ν $(\nu > 0)$ the k-th positive zero of the cylinder function $C_\nu(x) = J_\nu(x)\cos\alpha - Y_\nu(x)\sin\alpha$, $0 \leqslant \alpha < \pi$, has the asymptotic expansion

$$j_{\nu\kappa} = \nu + \gamma_\kappa \nu^{1/3} + \frac{3}{10}\gamma_\kappa^2 \nu^{-1/3} + \mathcal{O}(\nu^{-1})$$

where $\kappa = k - \alpha/\pi$, $\gamma_\kappa = -a_\kappa 2^{-1/3}$ and a_κ is the k-th negative zero of the function $Ai(x)\cos\alpha + Bi(x)\sin\alpha$ and $Ai(x)$, $Bi(x)$ denote the Airy functions of the first and the second kind, respectively [1]. We prove that the sum of the first three terms of the asymptotic expansion gives an upper bound for $j_{\nu\kappa}$, provided $\gamma_\kappa \geqslant \sqrt[3]{35/4} = 2.0606427\ldots$ or $\kappa \geqslant \kappa_0 = 1.13019788\cdots = 2 - \alpha_0/\pi$ where α_0 is determined by the equation $\cos\alpha_0 Ai\left(-\sqrt[3]{35/4}\right) + \sin\alpha_0 Bi\left(-\sqrt[3]{35/4}\right) = 0$. This result covers the cases $j_{\nu 2}, j_{\nu 3}, \ldots$ and $y_{\nu 2}, y_{\nu 3}, \ldots$, for all $\nu > 0$. The main tool used is the well-known Watson formula for $dj_{\nu\kappa}/d\nu$.

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