

SOME HILBERT SPACE CHARACTERIZATIONS AND BANACH SPACE INEQUALITIES

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Abstract. It is well known that if X is a normed linear space with dimension not less than three such that the radial projection from X onto the closed unit ball is nonexpansive, then X must be an inner product space. Using this fact, we are able to give a characterization of Hilbert spaces. Two other Hilbert space characterizations and some Banach space inequalities are established via duality maps.

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REFERENCES

- [1] F. E. BROWDER, *Convergence of approximants to fixed points of nonexpansive nonlinear maps in Banach spaces*, Arch. Rational Mech. Acad. **24** (1967), 82–90.
- [2] R. E. BRUCK, *The iterative solution of the equation $y \in x + Tx$ for a monotone operator T in Hilbert space*, Bull. Amer. Math. Soc. **79** (1973), 1258–1261.
- [3] J. CIORANESCU, *Geometry of Banach Spaces—Duality Mappings and Nonlinear Problems*, Mathematics and Its Applications **62**, Kluwer Academic Publisher, Dordrecht, The Netherlands, 1990.
- [4] D. G. DEFIGUEIREDO AND L. A. KARLOVITZ, *On the radial projection in normed spaces*, Bull. Amer. Math. Soc. **73** (1967), 364–368.
- [5] J. DIESTEL, *Geometry of Banach Spaces—Selected Topics*, Lect. Notes in Mathematics **485**, Springer-Verlag, New-York, 1970.
- [6] B. HARPERN, *Fixed points of nonexpanding maps*, Bull. Amer. Math. Soc. **73** (1967), 957–961.
- [7] K. KIM AND S. S. SHIN, *On approximating fixed points in Banach spaces*, Preprint.
- [8] J. A. PARK, *Mann-iteration process for the fixed point of strictly pseudocontractive mapping in some Banach spaces*, J. Korean Math. Soc. **31** (1994), 333–337.
- [9] J. A. PARK, *Mann-iteration process to the solution of $y = x + Tx$ for an accretive operator T in some Banach spaces*, Comm. Korean Math. Soc. **9** (1994), 819–823.
- [10] S. P. SINGH AND B. WATSON, *On approximating fixed points*, Proc. Symp. Pure Math. **45** (1986), Part 2, 393–395.
- [11] H. K. XU, *Inequalities in Banach spaces with applications*, Nonlinear Analysis **16** (1991), 1127–1136.
- [12] H. K. XU, *Approximating curves of nonexpansive nonself mappings in Banach spaces*, C. R. Acad. Sci., Paris, Ser. I, to appear.