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OPERATOR FUNCTIONS IMPLYING GENERALIZED FURUTA INEQUALITY *

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Abstract. As further extensions of the main result in [11], we show the following result.

Let $A \geqslant B \geqslant 0$ *with* $A > 0$. *For each* $t \in [0,1]$ *and* $p \geqslant t$, *the following* (i) *and* (ii) *hold for a fixed real number* q *and they are mutually equivalent:*

(i) *if* $q \geqslant 0$, *then*

$$G_{p,q,t}(A,B,r,s) = A^{\frac{-r}{2}}\{A^{\frac{r}{2}}(A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^sA^{\frac{r}{2}}\}^{\frac{q-t+r}{(p-t)s+r}}A^{\frac{-r}{2}}$$

is decreasing for $r \geqslant t$ *and* $s \geqslant 1$ *such that* $(p-t)s \geqslant q-t$.

(ii) *if* $p \geqslant q$, *then*

$$G_{p,q,t}(A,B,r,s) = A^{\frac{-r}{2}}\{A^{\frac{r}{2}}(A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^sA^{\frac{r}{2}}\}^{\frac{q-t+r}{(p-t)s+r}}A^{\frac{-r}{2}}$$

is decreasing for $s \geqslant 1$ *and* $r \geqslant \max\{t, t-q\}$.

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