OPERATOR FUNCTIONS IMPLYING GENERALIZED FURUTA INEQUALITY

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Abstract. As further extensions of the main result in [11], we show the following result.
Let \( A \geq B \geq 0 \) with \( A > 0 \). For each \( t \in [0,1] \) and \( p \geq t \), the following (i) and (ii) hold for a fixed real number \( q \) and they are mutually equivalent:
(i) if \( q \geq 0 \), then
\[
G_{p,q,t}(A, B, r, s) = A^{\frac{t}{p-t}} \{ A^{\frac{t}{q-t}} (A^{\frac{t}{r-t}} B^p A^{\frac{t}{s-t}}) A^{\frac{t}{q-t}} \}^{\frac{q-t}{p-t}} A^{\frac{p-t}{p-t}}
\]
is decreasing for \( r \geq t \) and \( s \geq 1 \) such that \( (p-t)s \geq q-t \).
(ii) if \( p \geq q \), then
\[
G_{p,q,t}(A, B, r, s) = A^{\frac{t}{p-t}} \{ A^{\frac{t}{q-t}} (A^{\frac{t}{r-t}} B^p A^{\frac{t}{s-t}}) A^{\frac{t}{q-t}} \}^{\frac{q-t}{p-t}} A^{\frac{p-t}{p-t}}
\]
is decreasing for \( s \geq 1 \) and \( r \geq \max \{t, t-q\} \).

Key words and phrases: Löwner-Heinz inequality, Furuta inequality, chaotic order.

REFERENCES

[6] T. Furuta, If \( A \geq B \geq 0 \) assures \( (B^r A^p B^r)^{1/q} \geq B^{(p+2r)/q} \) for \( r \geq 0 \), \( p \geq 0 \), \( q \geq 1 \) with \( (1+2r)q \geq p+2r \), Proc. Amer. Math. Soc. 101 (1987), 85–88.