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OPERATOR FUNCTIONS IMPLYING GENERALIZED FURUTA INEQUALITY *

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Abstract. As further extensions of the main result in [11], we show the following result.

Let $A \geqslant B \geqslant 0$ with $A > 0$. For each $t \in [0,1]$ and $p \geqslant t$, the following (i) and (ii) hold for a fixed real number q and they are mutually equivalent:

(i) *if $q \geqslant 0$, then*

$$G_{p,q,t}(A,B,r,s) = A^{\frac{-r}{2}}\{A^{\frac{r}{2}}(A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^sA^{\frac{r}{2}}\}^{\frac{q-t+r}{(p-t)s+r}}A^{\frac{-r}{2}}$$

is decreasing for $r \geqslant t$ and $s \geqslant 1$ such that $(p-t)s \geqslant q-t$.

(ii) *if $p \geqslant q$, then*

$$G_{p,q,t}(A,B,r,s) = A^{\frac{-r}{2}}\{A^{\frac{r}{2}}(A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^sA^{\frac{r}{2}}\}^{\frac{q-t+r}{(p-t)s+r}}A^{\frac{-r}{2}}$$

is decreasing for $s \geqslant 1$ and $r \geqslant \max\{t, t-q\}$.

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REFERENCES

[1] T. ANDO, *On some operator inequalities*, Math. Ann. **279** (1987), 157–159.
[2] T. ANDO AND F. HIAI, *Log-majorization and complementary Golden-Thompson type inequalities*, Linear Algebra Appl. **197, 198** (1994), 113–131.
[3] M. FUJII, *Furuta's inequality and its mean theoretic approach*, J. Operator Theory **23** (1990), 67–72.
[4] M. FUJII, T. FURUTA AND E. KAMEI, *Furuta's inequality and its application to Ando's theorem*, Linear Algebra Appl. **179** (1993), 161–169.
[5] M. FUJII AND E. KAMEI, *Mean theoretic approach to the grand Furuta inequality*, Proc. Amer. Math. Soc. **124** (1996), 2751–2756.
[6] T. FURUTA, *$A \geqslant B \geqslant 0$ assures $(B^rA^pB^r)^{1/q} \geqslant B^{(p+2r)/q}$ for $r \geqslant 0$, $p \geqslant 0$, $q \geqslant 1$ with $(1+2r)q \geqslant p+2r$*, Proc. Amer. Math. Soc. **101** (1987), 85–88.
[7] T. FURUTA, *Elementary proof of an order preserving inequality*, Proc. Japan Acad. **65** (1989), 126.
[8] T. FURUTA, *Two operator functions with monotone property*, Proc. Amer. Math. Soc. **111** (1991), 511–516.
[9] T. FURUTA, *Applications of order preserving operator inequalities*, Operator Theory: Advances and Applications **59** (1992), 180-190.
[10] T. FURUTA, *Extension of the Furuta inequality and Ando-Hiai log-majorization*, Linear Algebra Appl. **219** (1995), 139–155.
[11] T. FURUTA AND D. WANG, *A decreasing operator function associated with the Furuta inequality*, preprint.
[12] E. KAMEI, *A satellite to Furuta's inequality*, Math. Japon. **33** (1988), 883–886.
[13] K. TANAHASHI, *Best possibility of the Furuta inequality*, Proc. Amer. Math. Soc. **124** (1996), 141–146.

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