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A NOTE ON SIMULTANEOUSLY DIAGONALIZABLE MATRICES

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Abstract. Consider the functional $f(U) = \sum_{i=1}^{n} \max_j \{(U^T M_j U)_{ii}\}$ over orthogonal matrices U, where the collection of n-by-n symmetric matrices M_j are pairwise commutative, and thus simultaneously diagonalizable. Selecting an orthogonal matrix U which maximizes $f(U)$ has applications in adaptive optics. A proof is given here that any orthogonal matrix Q which simultaneously diagonalizes the M_j maximizes the function f.

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