

## A NOTE ON SIMULTANEOUSLY DIAGONALIZABLE MATRICES

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*Abstract.* Consider the functional  $f(U) = \sum_{i=1}^n \max_j \{(U^T M_j U)_{ii}\}$  over orthogonal matrices  $U$ , where the collection of  $n$ -by- $n$  symmetric matrices  $M_j$  are pairwise commutative, and thus simultaneously diagonalizable. Selecting an orthogonal matrix  $U$  which maximizes  $f(U)$  has applications in adaptive optics. A proof is given here that any orthogonal matrix  $Q$  which simultaneously diagonalizes the  $M_j$  maximizes the function  $f$ .

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### REFERENCES

- [1] A. BERMAN AND R. J. PLEMMONS, *Nonnegative Matrices in the Mathematical Sciences*, 2nd Edition, SIAM Press Classics Series no. 9, Philadelphia, 1994.
- [2] A. BUNSE-GERSTNER, R. BYERS, AND V. MEHRMANN, *Numerical methods for simultaneous diagonalization*, SIAM J. Matrix Anal. and Applic., **14** (1993), 927–949.
- [3] B. D. FLURY AND B. E. NEUENSCHWANDER, *Simultaneous diagonalization algorithms with applications in multivariate statistics*, in Approximation and Computation (R. Zahar, ed.), pp. 179–205, Basel: Birkhauser, 1995.
- [4] J. W. HARDY, *Adaptive optics*, Scientific American, **270**, no. 6, June (1994), 60–65.
- [5] R. HORN AND C. JOHNSON, *Matrix Analysis*, Cambridge University Press, Cambridge, England, 1985.
- [6] J. NELSON, *Reinventing the telescope*, Popular Science, 85, January (1995), 57–59.
- [7] R. A. SMITH, *Seeing the universe through new eyes*, National Geographic Magazine, **185**, no. 1, January (1994), 2–41.