

## A REFINEMENT OF A THEOREM OF PAUL TURAN CONCERNING POLYNOMIALS

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*Abstract.* In this paper we establish certain sharp results concerning the maximum modulus of the polar derivative of a polynomial  $P(z)$  with restricted zeros. Our results generalize and refine some results of Turán, Malik, Govil and others.

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