

OPERATOR FUNCTIONS ASSOCIATED WITH THE GRAND FURUTA INEQUALITY

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Abstract. We discuss the monotonicity of operator functions associated with the grand Furuta inequality, some of which are considered under the chaotic order $\log A \geq \log B$. In some restricted cases, several known operator inequalities related to the Furuta inequality will appear as corollaries of our results.

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