

## NORMS AND DETERMINANTS OF PRODUCTS OF LOGARITHMIC FUNCTIONS OF POSITIVE SEMI-DEFINITE OPERATORS

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*Abstract.* Let  $A, B$  be bounded positive semi-definite operators (matrices) on a Hilbert space. We will show

$$\|\log(1 + A) \log(1 + B)\| \leq \{\log(1 + \|AB\|^{\frac{1}{2}})\}^2,$$

and

$$\|\log(1 + B) \log(1 + A) \log(1 + B)\| \leq \{\log(1 + \|BAB\|^{\frac{1}{3}})\}^3.$$

Further we will prove the corresponding determinantal inequalities.

*Mathematics subject classification (1991):* Primary 47A30; Secondary 15A15, 15A45.

*Key words and phrases:* Norm, determinant, positive semi-definite operator, logarithmic function, operator monotone function, majorization.

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