

NORMS AND DETERMINANTS OF PRODUCTS OF LOGARITHMIC FUNCTIONS OF POSITIVE SEMI-DEFINITE OPERATORS

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Abstract. Let A, B be bounded positive semi-definite operators (matrices) on a Hilbert space. We will show

$$\|\log(1 + A) \log(1 + B)\| \leq \{\log(1 + \|AB\|^{\frac{1}{2}})\}^2,$$

and

$$\|\log(1 + B) \log(1 + A) \log(1 + B)\| \leq \{\log(1 + \|BAB\|^{\frac{1}{3}})\}^3.$$

Further we will prove the corresponding determinantal inequalities.

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REFERENCES

- [1] T. ANDO, *Majorizations, and inequalities in matrix theory*, Linear Algebra Appl. **199** (1994), 17–67.
- [2] R. BHATIA, *Matrix Analysis*, Springer-Verlag, New York, 1996.
- [3] T. FURUTA, *Norm inequalities equivalent to Löwner-Heinz theorem*, Rev. Math. Phys. **1** (1989), 135–137.
- [4] F. HIAI, *Log-majorizations and norm inequalities for exponential operators*, in *Linear Operators*, Banach center Publications **38** (1997), 119–181.