

ON A CONJECTURE ON THE CLOSEST NORMAL MATRIX

ANDERS BARRLUND

Abstract. Let A be a complex $n \times n$ matrix and let \mathcal{N}_n be the set of normal $n \times n$ matrices. A conjecture is that

$$\|A - \mathcal{N}_n\|_F^2 \leq \frac{n-1}{n} \text{dep}^2(A),$$

where $\text{dep}^2(A) = \|A\|_F^2 - \sum_{i=1}^n \lambda_i^2(A)$ and $\lambda_i(A), i = 1, \dots, n$ are the eigenvalues of A . We prove that the conjecture is correct for all even n and for $n = 3, 5, 7$. However, for the dimensions, $n = 3, 5, 6, 7$, and presumably also other problem dimensions it is possible to derive sharper bounds. We also prove a bound for odd n which converges to the bound in the conjecture when n tends to infinity. The main idea in the proofs is to use LP problems with constraints based on different ways to approximate A with normal matrices.

Mathematics subject classification (1991): 15A45.

Key words and phrases: Normal matrix, LP-problem.

REFERENCES

- [1] L. Elsner and Kh. D. Ikramov, *Towards a proof of László's conjecture*, private communication with L. Elsner (hand-written letter from 1996).
- [2] P. E. Gill, W. Murray and M. H. Wright, *Numerical Linear Algebra and Optimization* Vol 1, Addison-Wesley, 1990.
- [3] Kh. D. Ikramov, *On normal completions of triangular matrices*, Doklady Akademii Nauk, 351 (1996), pp. 1–2 (in Russian).
- [4] Kh. D. Ikramov, *On normal dilations of triangular matrices*, Mathematical notes, 60, N6(1996), pp. 861–872 (in Russian).
- [5] L. László, *Upper bounds for the best normal approximation*, *Mathematica Pannonica* 9, 1 (1998), pp. 121–129.