ON A CONJECTURE ON THE CLOSEST NORMAL MATRIX

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Abstract. Let \( A \) be a complex \( n \times n \) matrix and let \( \mathcal{N}_n \) be the set of normal \( n \times n \) matrices. A conjecture is that

\[
\| A - \mathcal{N}_n \|_F^2 \leq \frac{n - 1}{n} \text{dep}^2(A),
\]

where \( \text{dep}^2(A) = \| A \|_F^2 - \sum_{i=1}^{n} \lambda_i^2(A) \) and \( \lambda_i(A), i = 1, \ldots, n \) are the eigenvalues of \( A \). We prove that the conjecture is correct for all even \( n \) and for \( n = 3, 5, 7 \). However, for the dimensions, \( n = 3, 5, 6, 7 \), and presumably also other problem dimensions it is possible to derive sharper bounds. We also prove a bound for odd \( n \) which converges to the bound in the conjecture when \( n \) tends to infinity. The main idea in the proofs is to use LP problems with constraints based on different ways to approximate \( A \) with normal matrices.

Key words and phrases: Normal matrix, LP-problem.

REFERENCES