

OPTIMIZATION OF SCHUR-CONVEX FUNCTIONS

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Abstract. In this paper, we show that Schur-convex functions share some important properties with the ordinary convex functions. We apply special properties of Schur-convex functions to establish some inequalities for the generalized power means that include many well-known classical analytic inequalities as special cases.

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