INEQUALITIES FOR POLYNOMIALS WITH A PRESCRIBED ZERO

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Abstract. For a polynomial $p(z)$ of degree $n$, having a zero of order $k \ (\geq 1)$ at $\beta$, we have obtained

$$\max_{|z|=1} \left| \frac{p(z)}{(z-\beta)^k} \right| \leq \left( \frac{n-k+1}{1+|\beta|} \right)^k \max_{1 \leq i \leq n-k+1} |p(v'_i)|,$$

$v'_1, v'_2, \ldots, v'_{n-k+1}$ being the roots of $z^{n-k+1} + e^{i\gamma(n-k+1)} = 0$, with $\gamma = \arg \beta \ (\gamma = 0$ for $\beta = 0)$, thereby extending the previously known estimate (i.e. $\max_{|z|=1} \left| \frac{p(z)}{z-\beta} \right| \leq \frac{n}{1+|\beta|} \max_{1 \leq i \leq n} |p(z_i)|$, $\beta \geq 0$, $z_1, z_2, \ldots, z_n$ being the roots of $z^n + 1 = 0$).

Key words and phrases: Polynomials, prescribed zero, roots.

REFERENCES