

ON THE OSCILLATION OF SECOND ORDER NONLINEAR DIFFERENCE EQUATIONS

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Abstract. We shall discuss two powerful techniques, namely, the averaging method, and the inequalities method, which have been used for quite some time to establish the oscillations of second order differential equations, whereas their use in the study of difference equations is recent and deserves more attention.

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