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OSCILLATION OF EVEN ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS WITH DAMPING

Y. ŞAHINER YILMAZ AND A. ZAFER

Abstract. Oscillation criteria for even order differential equations of the following form

$$z^{(n)}(t) + p(t)\phi(z^{(n-1)}(t)) + q(t)|x(\sigma(t))|^{\alpha}\operatorname{sgn}[x(\sigma(t))] = 0,$$

where

$$z(t) = x(t) + a(t)x(\tau(t)), \quad \alpha > 0, \quad \text{and } n \text{ is even}$$

are obtained via comparison with second order differential inequalities. It is shown that existence of no eventually positive solution of a certain second order delay differential inequality is sufficient for every solution $x(t)$ of the above equation to be oscillatory.

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