Mathematical
Inequalities
& Applications
Volume 1, Number 4 (1998), 463–480

DECOMPOSITION OF HOMOGENEOUS MEANS AND CONSTRUCTION OF SOME METRIC SPACES

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Abstract. Any (positively) homogeneous mean on $(0,\infty)^2$ can be decomposed multiplicatively into the arithmetic mean A and a one–place function, called A*–index function*. Index functions characterize a homogeneous mean in many respects, and their graphs are suitable for geometrical comparisons of several properties of homogeneous means. Moreover, index functions can facilitate proofs of inequalities between different types of homogeneous means. With the aid of A–index functions, some metrics are introduced in the set of homogeneous means.

Mathematics subject classification (1991): 26D07.
Key words and phrases: Homogeneous mean, index function of a mean, decomposition of a mean, complete metric space of homogeneous means.

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Paper MIA-01-45