

## MULTI-DIMENSIONAL INTEGRAL INEQUALITIES OF THE WIRTINGER-TYPE

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*Abstract.* In this paper some new Wirtinger-type integral inequalities involving many functions of many variables are established. These on the one hand improve existing results in the subject concerned and on the other hand can serve as generators of other integral inequalities of such type.

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