

MIXED MEANS AND HARDY'S INEQUALITY

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Abstract. Integral means of arbitrary order, with power weights, and their companion means are introduced and related mixed-means inequalities are derived. These results are then used in proving inequalities of Hardy and Levin-Cochran-Lee type. Also, new proofs of Hardy and Carleman inequality for finite and infinite series are given by using discrete mixed-means.

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REFERENCES

- [1] M. Alić and J. Pečarić, *Inequalities related to Hardy's and Levin's*, Rad Hrvatske akad. znan. umj. mat. [467] **11** (1994), 111–117.
- [2] N. G. De Bruijn, *Carleman's inequality for finite series*, Nederl. Akad. Wet. Amsterdam. Proc. Ser. A. **44** (1963), 505–514 (= *Indagationes Mathematicae* **34**).
- [3] P. S. Bullen, *A Chapter on Inequalities*, Math. Medley **21**, No. 2 (1993), 48–69.
- [4] P. S. Bullen, *Inequalities Due to T. S. Nanjundiah*, manuscript
- [5] J. A. Cochran and C.-S. Lee, *Inequalities related to Hardy's and Heinig's*, Math. Proc. Cambridge Phil. Soc. **96** (1984), 1–7.
- [6] G. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, second edition, Cambridge University Press, Cambridge, 1967.
- [7] F. Holland, *On a mixed arithmetic-mean, geometric-mean inequality*, Mathematics Competitions **5** (1992), 60–64.
- [8] K. Kedlaya, *Proof of a Mixed Arithmetic-Mean, Geometric-Mean Inequality*, Amer. Math. Monthly, **101** (1994), 355–357.
- [9] V. Levin, *O neravenstvah III: Neravenstva, vpolnjaemie geometričeskim srednim neotricatel'noi funkcii*, Math. Sbornik **4** (**46**) (1938), 325–331.
- [10] E.R. Love, *Inequalities related to those of Hardy and of Cochran and Lee*, Math. Proc. Cambridge Phil. Soc. **99** (1986), 395–408.
- [11] A. Lupaş, *Asupra problemei 579 (1901) din Gazeta Matematica*, Gaz. Mat. (Bucharest) **81** (1976), 281–286.
- [12] T. Matsuda, *An Inductive Proof of a Mixed Arithmetic-Geometric Mean Inequality*, Amer. Math. Monthly **102** (1995), 634–637.
- [13] D. S. Mitrinović, J. E. Pečarić and A. M. Fink, *Inequalities Involving Functions and Their Integrals and Derivatives*, Kluwer Academic Publishers, 1991.
- [14] B. Mond and J. Pečarić, *A Mixed means Inequality*, Austral. Math. Soc. Gazette, **23** (1996), No. 2, 67–70.
- [15] B. Mond and J. Pečarić, *A Mixed Arithmetic-Mean-Harmonic-Mean Matrix Inequality*, Linear Algebra Appl. **237/238** (1996), 449–454.
- [16] B. Mond and J. Pečarić, *Mixed means inequalities for positive linear operators*, Austral. Math. Soc. Gazette, **23** (1996), No. 5, 198–200.
- [17] T. S. Nanjundiah, *Sharpening some classical inequalities*, Math. Student, **20** (1952), 24–25.

- [18] W. Rudin, *Real and Complex Analysis*, McGraw–Hill, 1970.
- [19] Gou–Sheng Yang, Yu–Jen Lin, *On companion inequalities related to Heinig’s*, Tamkang Journal of Math., **22** (1991), No. 4, 313–322.
- [20] H. S. Wilf, *On finite sections of the classical inequalities*. Nederl. Akad. Wet. Amsterdam. Proc. Ser. A. **65** (1962), 340–342 (Indagationes Mathematicae **24**).