INEQUALITIES FOR THE MINIMAL EIGENVALUE OF THE LAPLACIAN IN AN ANNULUS

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Abstract. We discuss the behavior of the minimal eigenvalue $\lambda$ of the Dirichlet Laplacian in the domain $D_1 \setminus D_2 := D$ (an annulus) where $D_1$ is a circular disc and $D_2 \subset D_1$ is a smaller circular disc. It is conjectured that the minimal eigenvalue $\lambda$ has a maximum value when $D_2$ is a concentric disc. If $h$ is a displacement of the center of the disc $D_2$ and $\lambda(h)$ is the corresponding minimal eigenvalue, then $\frac{d\lambda(h)}{dh} < 0$ so that $\lambda(h)$ is minimal when $\partial D_2$ touches $\partial D_1$, where $\partial D$ is the boundary of $D$. Numerical results are given to back the conjecture. Upper and lower bounds are given for $\lambda(h)$.

Key words and phrases: Inequalities, estimation of eigenvalues, perturbation theory.

REFERENCES