GLOBAL LIMITING EMBEDDINGS OF
LOGARITHMIC BESSEL POTENTIAL SPACES

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Abstract. The paper is a continuation of [EGO II-IV], where it was shown that Bessel potential spaces $H^\sigma Y(\mathbb{R}^n)$, modelled upon appropriate generalized Lorentz-Zygmund spaces $Y(\mathbb{R}^n)$ may be embedded into Orlicz spaces $L_\Phi(\Omega)$, where $\Phi(t) = \exp(\exp(\ldots \exp t^\nu) \ldots))$ for large t, $\nu > 0$, and Ω is a subset of $\mathbb{R}^n$ with finite volume. Using weighted Hardy inequalities, we modify the Young function Φ near the origin so that the above embedding holds with Ω replaced by $\mathbb{R}^n$. The resulting Young function dominates globally the Young function $\Psi(t) = t^q$, $t > 0$, for q sufficiently large and consequently, $H^\sigma Y(\mathbb{R}^n) \hookrightarrow L^q(\mathbb{R}^n)$. We also obtain an estimate of the norms of the last embeddings which is sharp in their dependence upon q provided that q is large enough.

REFERENCES

[A]   R. A. ADAMS, *Sobolev spaces*, Academic Press, New York, 1975.

[AS]   N. ARONSZAJN AND K. T. SMITH, *Theory of Bessel potentials*, Part I, Ann. Inst. Fourier **11** (1961), 385–475.

[BR]   C. BENNETT AND K. RUDNICK, *On Lorentz-Zygmund spaces*, Dissertationes Math. **175** (1980), 1–72.

[BS]   C. BENNETT AND R. SHARPLEY, *Interpolation of operators*, Pure Appl. Math. 129, Academic Press, New York, 1988.

[BW]   H. BRÉZIS AND S. WAINGER, *A note on limiting cases of Sobolev embeddings and convolution inequalities*, Comm. Partial Differential Equations **5** (1980), 773–789.

[EE]   D. E. EDMUNDS AND W. D. EVANS, *Orlicz and Sobolev spaces on unbounded domains*, Proc. Roy. Soc. London Ser. A **342** (1975), 373–400.

[EGO I] D. E. EDMUNDS, P. GURKA AND B. OPIC, *Double exponential integrability of convolution operators in generalized Lorentz-Zygmund spaces*, Indiana Univ. Math. J. **44** (1995), 19–43.

[EGO II] D. E. EDMUNDS, P. GURKA AND B. OPIC, *Double exponential integrability, Bessel potentials and embedding theorems*, Studia Math. **115** (1995), 151–181.

[EGO III] D. E. EDMUNDS, P. GURKA AND B. OPIC, *Sharpness of embeddings in logarithmic Bessel-potential spaces*, Proc. Roy. Soc. Edinburgh **126A** (1996), 995–1009.

[EGO IV] D. E. EDMUNDS, P. GURKA AND B. OPIC, *On embeddings of logarithmic Bessel potential spaces*, J. Functional Anal. **146** (1997), 116–150.

[EGO V] D. E. EDMUNDS, P. GURKA AND B. OPIC, *Norms of embeddings of logarithmic Bessel potential spaces*, Proc. Amer. Math. Soc. (to appear).

[EK]   D. E. EDMUNDS AND M. KRBEC, *Two limiting cases of Sobolev imbeddings*, Houston J. Math. **21** (1995), 119–128.

[EOP]   W. D. EVANS, B. OPIC AND L. PICK, *Interpolation of operators on scales of generalized Lorentz–Zygmund spaces*, Math. Nachr. **182** (1996), 127–181.

[FLS] N. Fusco, P. L. Lions and C. Sbordone, *Sobolev imbedding theorems in borderline cases*, Proc. Amer. Math. Soc. **124** no. 2 (1996), 561–565.

[O] R. O'Neil, *Convolution operators and $L(p,q)$ spaces*, Duke Math. J. **30** (1963), 129–142.

[OK] B. Opic and A. Kufner, *Hardy-Type Inequalities*, Pitman Research Notes in Mathematics 219, Longman, Harlow, 1990.

[OP] B. Opic and L. Pick, *On generalized Lorentz-Zygmund spaces* (to appear).

[T] N. S. Trudinger, *On imbeddings into Orlicz spaces and some applications*, J. Math. Mech. **17** (1967), 473–484.

[Z] W. Ziemer, *Weakly differentiable functions*, Graduate Texts in Math. 120, Springer, Berlin, 1989.

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