GLOBAL LIMITING EMBEDDINGS OF LOGARITHMIC BESSEL POTENTIAL SPACES

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Abstract. The paper is a continuation of [EGO II-IV], where it was shown that Bessel potential spaces \( H^\sigma Y(\mathbb{R}^n) \), modelled upon appropriate generalized Lorentz-Zygmund spaces \( Y(\mathbb{R}^n) \) may be embedded into Orlicz spaces \( L_\Phi(\Omega) \), where \( \Phi(t) = \exp(\exp(\ldots \exp t^\nu)\ldots) \) for large \( t, \nu > 0 \), and \( \Omega \) is a subset of \( \mathbb{R}^n \) with finite volume. Using weighted Hardy inequalities, we modify the Young function \( \Phi \) near the origin so that the above embedding holds with \( \Omega \) replaced by \( \mathbb{R}^n \). The resulting Young function dominates globally the Young function \( \Psi(t) = t^q, t > 0, \) for \( q \) sufficiently large and consequently, \( H^\sigma Y(\mathbb{R}^n) \hookrightarrow L^q(\mathbb{R}^n) \). We also obtain an estimate of the norms of the last embeddings which is sharp in their dependence upon \( q \) provided that \( q \) is large enough.

Key words and phrases: Generalized Lorentz-Zygmund spaces, logarithmic Bessel potential spaces, Orlicz spaces, embeddings.

REFERENCES


