ON THE ROOTS OF LACUNARY POLYNOMIALS

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Abstract. We prove estimates for the roots of lacunary polynomials. They are deduced from the study of the equation \( x^n - x^{n-1} = a \), where \( n \in \mathbb{N} \), \( n \geq 2 \), \( a > 0 \).

Let \( P(X) = \sum_{i=0}^{m} a_i X^i \) be a nonzero complex polynomial. We are interested to find bounds for the absolute values of the roots of \( P \) in function of the coefficients \( a_i \). Such estimates were obtained by P. Montel [4] and more recently by M. Mignotte [3]. Other bounds are given by evaluations valid for arbitrary polynomials (good references are to be found, for example, in the monographs of P. Henrici [2] and P. Borwein–T. Erdélyi [1]).

In this paper we obtain such bounds in the following way:

To the polynomial \( P \) we associate convenient \( a > 0 \) and \( n \in \mathbb{N} \setminus \{0, 1\} \), with \( a = a(a_0, a_1, \ldots, a_m) > 0 \). Therefore we obtain bounds for the unique root \( \xi > 1 \) of the equation \( x^n - x^{n-1} = a \). This allows us to describe bounds for the roots of the original polynomial \( P \).

In particular, this method gives good estimates for the case of lacunary polynomials, i.e. for polynomials with a certain number of consecutive zero coefficients.

Key words and phrases: roots of polynomials, lacunary polynomials.

REFERENCES