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ON THE ROOTS OF LACUNARY POLYNOMIALS

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Abstract. We prove estimates for the roots of lacunary polynomials. They are deduced from the study of the equation $x^n - x^{n-1} = a$, where $n \in \mathbf{N}$, $n \geqslant 2$, $a > 0$.

Let $P(X) = \sum_{i=0}^{m} a_i X^i$ be a nonzero complex polynomial. We are interested to find bounds for the absolute values of the roots of P in function of the coefficients a_i. Such estimates were obtained by P. Montel [4] and more recently by M. Mignotte [3]. Other bounds are given by evaluations valid for arbitrary polynomials (good references are to be found, for example, in the monographs of P. Henrici [2] and P. Borwein–T. Erdélyi [1]).

In this paper we obtain such bounds in the following way:

To the polynomial P we associate convenient $a > 0$ and $n \in \mathbf{N} \setminus \{0, 1\}$, with $a = a(a_0, a_1, \ldots, a_m) > 0$. Therefore we obtain bounds for the unique root $\xi > 1$ of the equation $x^n - x^{n-1} = a$. This allows us to describe bounds for the roots of the original polynomial P. In particular, this method gives good estimates for the case of lacunary polynomials, i.e. for polynomials with a certain number of consecutive zero coefficients.

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