

EXTENSIONS OF THE GEOMETRIC–ARITHMETIC MEANS INEQUALITY TO A DISC OF THE COMPLEX PLANE

RICHARD FOURNIER

Abstract. For complex numbers z_j with $|z_j - 1| \leq r$, $r < 1$, we consider the arithmetic mean $A_n := (1/n) \sum_{j=1}^n z_j$ and the geometric mean $G_n := \prod_{j=1}^n z_j^{1/n}$ and prove, amongst other results, that

$$\frac{1 - r^2}{|2 - A_n|} \leq |G_n| \leq \frac{|A_n|}{\sqrt{1 - r^2}}.$$

Mathematics subject classification (1991): 26D15, 30C45.

Key words and phrases: geometric and arithmetic means in the complex plane, starlike univalent functions.

REFERENCES

- [1] E. F. BECKENBACH AND R. BELLMAN, *Inequalities*, Springer-Verlag, Berlin, 1971.
- [2] P. L. DUREN, *Univalent functions*, Springer-Verlag, New-York, 1983.
- [3] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Inequalities*, Cambridge University Press, Cambridge, 1952.
- [4] D. S. MITRINOVIĆ, *Analytic inequalities*, Springer-Verlag, Berlin, 1970.
- [5] ST. RUSCHEWEYH, *Convolution in geometric function theory*, Les Presses de l'Université de Montréal, Montréal, 1982.
- [6] T. B. SHEIL-SMALL, *Convolution of convex functions*, J. London Math. Soc. **2** (1969), 483–492.