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MEASURES OF ALGEBRAIC SUMS OF SETS

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Abstract. A variety of measure–theoretic inequalities are derived for algebraic sum sets involving sets with fractal structure. The derivations are based on combinatorial inequalities which in turn are derived from canonical univariate algebraic inequalities for polynomials in noninteger powers. A systematic procedure is presented and some known results generalized.

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